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A COMPARISON OF METHODS FOR CALIBRATION AND USE OF MULTI-COMPON--ETC(U)
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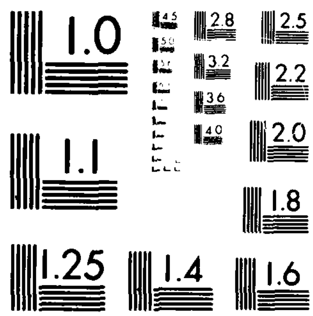

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LEVEL II

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A COMPARISON OF METHODS FOR CALIBRATION AND USE OF MULTI-COMPONENT STRAIN GAUGE WIND TUNNEL BALANCES

by
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A COMPARISON OF METHODS FOR CALIBRATION AND USE OF MULTI-COMPONENT STRAIN GAUGE WIND TUNNEL BALANCES

(COMPARAISON DE MÉTHODES D'ÉTALONNAGE ET D'UTILISATION DES BALANCES EXTENSOMÉTRIQUES DE SOUFFLERIE À PLUSIEURS COMPOSANTES)

by/par

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SUMMARY

A method is presented for calibration of strain-gauge balances which does not require that the components can be loaded independently. Applicable to both 'internal' and 'external' types of balance, the procedure uses a single varying calibration load to determine all linear and non-linear calibration coefficients. Constant 'secondary' loads on one or more components are unnecessary, although they may be used if desired.

The usual iterative solution of the second order balance equations is outlined, and an approximate non-iterative scheme is included for completeness, though not recommended. Two methods of accounting for dependency of the calibration coefficients on the signs of the component loads are presented.

A concept of 'buoyancy' is introduced to simplify the application of force balance tares, and a procedure for determining the component outputs for absolute zero load (the 'buoyant' offsets) is given. Balance data at a series of model attitudes are used to define these offsets, and also the coefficients in the equations defining the component load distribution of the tare weight at any attitude.

The topics covered are ideally suited to formulation and solution by matrix methods, which have been used throughout.

(français verso)

RÉSUMÉ

Il s'agit d'une méthode d'étalonnage de balances extensométriques qui n'exige pas le chargement séparé des composantes. Applicable aux balances 'internes' et 'externes', la méthode utilise une seule charge d'étalonnage variable pour déterminer tous les coefficients d'étalonnage, linéaires et quadratiques. Il n'est pas nécessaire d'appliquer des charges 'secondaires' constantes sur une ou plusieurs des composantes, mais on peut les employer si on veut.

La solution itérative courante des équations quadratiques de la balance est décrite, tout comme une autre méthode approximative, non itérative, qui n'est toutefois pas recommandée. Deux méthodes visant à compenser l'effet du signe des charges des composantes sur les coefficients d'étalonnage sont présentées.

Le concept de 'poussée' permet de simplifier l'application des tares d'équilibrage, et une méthode pour déterminer la sortie des composantes correspondant à une charge totale nulle (corrections de 'poussée') est donnée. Des données expérimentales pour différentes orientations de la balance servent à déterminer ces corrections ainsi que les coefficients des équations de répartition de la tare dans les composantes, pour une orientation quelconque de la balance.

Les sujets abordés sont particulièrement bien adaptés, quant à leur formulation et à leur solution, aux méthodes matricielles employées tout au cours de la présente étude.

CONTENTS

ILLUSTRATIONS

SYMBOLS

Symbol	Definition
a	coefficient of linear term in Equation 13
b	coefficient of quadratic term in Equation 13
[B]	non-linear coefficient matrix in approximate solution method (Eq. 46)
C	sensitivity and interaction coefficient
[C], [C1], [C2]	sensitivity and interaction coefficient matrices
[D]	diagonal matrix of primary sensitivity coefficients
F	component load
f	'load ratio' — fraction of applied calibrating load reacted by component
$\underline{F}$, $\underline{FL1}$, $\underline{FSQ1}$, $\underline{FSQ2}$, $\underline{FCP1}$, $\underline{FCP2}$, $\underline{FCP3}$, $\underline{FCP4}$	linear, load squared, and second order load cross product force vectors involving 'signed' and 'absolute' values of the component loads
$\underline{G1}$, $\underline{G2}$, $\underline{G3}$, [G], [G1]	vectors and matrices defining the component load distribution of model weight
K	constant load in calibration procedure
[L], [L1], [L2]	'load ratio' matrices (no constant loads)
[L*], [L1*], [L2*]	'load ratio' matrices when constant loads are present
m	component tare outputs in data system units
m	total number of sensitivity and interaction coefficients per component
n	number of components
p	number of non-linear interaction coefficients per component
P	variable calibrating load
R	component output reading

SYMBOLS

Symbol	Definition
r_o	intercept in Equation 13
[S]	matrix of curve-fit coefficients (linear and non-linear)
[T], [T1]	matrices of trigonometric functions of balance pitch and roll angles
W	tare weight
W_x, W_y, W_z	balance-axis components of tare weight (see Fig. 1)
[X], [X1], [X2]	normalized interaction coefficient matrices
[XL1], [XL2], [XSQ1], [XSQ2], [XCP1], [XCP2], [XCP3], [XCP4]	linear, load squared, and load cross product interaction coefficient matrices involving both 'signed' and 'absolute' values of the component loads
x_o, y_o, z_o	centre of gravity location of tare mass (see Fig. 1)
α	balance pitch angle
$\underline{\delta}$	vector of convergence limits in non-linear solution for 'tares'
Δ	matrix determinant
$\underline{\Delta}$	vector of non-linear effects in the balance equations (Eq. 42)
ϕ	balance roll angle

Superscripts

N	net
o	'buoyant' condition
T	total
*	non-linear vector or matrix of component loads
~	approximate value, ignoring interactions
$[\ ]^T$	transpose of matrix

Subscripts

i	'interacted upon' component
j	primary 'interacting' component
k	secondary 'interacting' component
z	loading case number

A COMPARISON OF METHODS FOR CALIBRATION AND USE OF MULTI-COMPONENT STRAIN GAUGE WIND TUNNEL BALANCES

1.0 INTRODUCTION

Extensive amounts of wind tunnel testing time are devoted to measurement of the static aerodynamic forces and moments acting on the test model[1]. Careful calibration of the balances used to measure these loads is necessary to obtain the best possible accuracy, the objective being the determination of the constants in the equations chosen to represent the actual behaviour of the balance. In a test environment these equations are solved to provide the forces and moments corresponding to the recorded measurements. Methods for balance calibration and data reduction are discussed in this report.

Depending on their location, wind tunnel balances can be classified into two types: 'external' balances which are located outside the model and test section, and 'internal' balances which are located inside the model or its supports, or may be integral with the model (or a portion of it) or the support. The 'external' type is generally regarded as being synonymous with the 'mechanical' balances with which low speed wind tunnels are usually equipped. In these, the total aerodynamic force and moment are resolved into components with the aid of kinematic mechanisms consisting of links which are considered undeformable[2], and it is possible to eliminate almost completely interactions between the components. However, as the balance is outside the test section the supports connecting the model to it are acted upon by aerodynamic forces and these effects, together with the interaction (interference) between the model and the supports, have to be taken into account when determining the true aerodynamic forces and moments acting on the model. The difficulty of correcting for these effects when the magnitudes increase at higher test velocities makes the 'internal' type of balance more suitable for high speed wind tunnel testing.

'Internal' balances enable the influence of the support to be greatly reduced. When the balance is located inside the model itself the loads acting on the support are not measured, although the interference effect of the support is still reflected in the measured model forces. Such balances find greatest application in testing at high subsonic, transonic, and supersonic velocities. As wind tunnels having operating envelopes covering one or more of these regimes typically have relatively small test sections, the models must be small also and 'internal' balances are consequently synonymous with strain gauge measurement techniques. In such balances the kinematic hinges of the 'mechanical' type are replaced by elastic hinges, thus converting the 'metric' (model) side of the balance into a kind of floating frame connected to the ground side, (i.e. the model support), by a statically determined system of links. By measuring the reactions in these links with the aid of strain-gauged elastic measuring elements, the components of the load applied to the system can be determined as functions of the strains in one or more elastic elements. By suitable design of the elastic elements the strain caused by the component of load or moment to be measured can be made to be very much greater than that caused by any other component of load. However, as opposed to 'mechanical' balances, with those of the strain gauge type it is usually not possible to eliminate completely the interactions between the various components, and these interactions must be determined by the calibration procedure.

In general the process of calibrating a balance can be defined as the acquisition of data relating the applied load to the strain gauge output from each component, for a sufficient number of independent loading situations to allow determination of all the constants in the equations chosen to define the behaviour of the balance. A common technique for the calibration of 'internal' strain gauge balances involves the application of loads which are reacted by a single component only[3,4]. However, such a procedure is not directly applicable to the 'external' type where it is either impossible to load the components independently, or to do so would represent a totally unrealistic loading situation in practice. The initial part of this report describes a calibration technique which has general applicability to both 'internal' and 'external' balances, and compares it with the more common technique mentioned.

The second part of the report is concerned with the determination of the aerodynamic loading experienced by a model in a wind tunnel test. To correctly compute these loads involves, firstly, solution of the balance equations to yield the gross loads on the model, and secondly, calculation and subtraction of the model weight tares. The common iterative approach to solution of the non-linear balance equations is described, and compared with an approximate method which does not require iteration. Also, as balance calibrations often treat positive and negative load behaviour separately, two methods of accounting for the dependency of the calibration and interaction coefficients upon the sign of the load are discussed.

The tare loads, in a balance axis system, are a function of both model weight and orientation. However, as a result of support deflections, the orientation is generally a function of the gross load, (unless it is sensed within the model itself), and determination of the aerodynamic loading normally involves iteration. A technique for the application of force balance tares, which avoids the iterative nature of the problem by introducing a concept of 'buoyant' tares, (tare readings for zero absolute load on each component), is described herein. The concept of 'buoyant' tares has the additional advantage of avoiding the problem of 'initial load effects' for systems in which the coefficients of second or higher order terms are non-zero, as the same origin, (i.e. zero absolute load), can be used for both calibration and use of the balance.

2.0 STRAIN GAUGE BALANCE CALIBRATION

While a primary objective in the design of strain gauge balances is the minimization of interactions between components, i.e. output on a component resulting from load applied to a different component, it is not generally possible to eliminate them completely. The interactions may be classified as either 'linear' or 'non-linear' according to whether they are functions of a single load component or a combination of load components. Typically the linear terms result from such things as construction errors (manufacturing tolerances and assembly misalignments for multi-part balances), improper positioning of the strain gauges, variations of gauge factor, and the electrical circuits; the non-linear terms are attributable to deflections. In cases where the design loads are relatively large in relation to the size of the balance the non-linear effects will be more significant, and it may be necessary to account for them in order to achieve an acceptable measurement accuracy. The decision of whether to calibrate a balance as a linear or non-linear system should be determined by whether or not the measurement accuracy required in the given application can be achieved by ignoring non-linear effects. If a linear calibration suffices there is a considerable saving in the effort required to perform the calibration, and also some computational simplification, although this latter point is relatively insignificant with modern computers.

For generality in describing procedures for the calibration of multi-component strain gauge balances, we will consider one designed to measure 'n' components of load and moment, and will assume that the outputs of the strain gauge bridge for each component are, as a result of interactions, functions of all component loads.

From Reference 3, the reading (strain gauge bridge output) for the 'i'th. balance component can be expressed as a polynomial function of the form:

$$\begin{aligned} R_i = {} & C_{i.1} F_1 + C_{i.2} F_2 + \cdots\cdots + C_{i.n} F_n \\ & + C_{i.11} F_1^2 + C_{i.22} F_2^2 + \cdots\cdots + C_{i.nn} F_n^2 \\ & + C_{i.12} F_1 F_2 + C_{i.13} F_1 F_3 + \cdots + C_{i.(n-1)n} F_{(n-1)} F_n \\ & + C_{i.111} F_1^3 + C_{i.222} F_2^3 + \cdots + C_{i.123} F_1 F_2 F_3 + \cdots \text{ad. inf.} \end{aligned} \tag{1}$$

The output of course depends upon the bridge excitation voltage, but it is usual to express all quantities in terms of unit voltage.

There will be 'n' equations of this type ($i = 1$ to n) in which the various constants are the required calibration coefficients. Practical experience with typical balances by a wide range of users has indicated that terms of third and higher degree can be ignored safely, and that second degree terms are nearly always small compared to the direct linear term in each equation. In what follows it will be assumed therefore that each component output is represented to sufficient accuracy by a second order polynomial of the form:

$$\begin{aligned} R_i = {} & C_{i.1} F_1 + C_{i.2} F_2 + \cdots\cdots + C_{i.n} F_n \\ & + C_{i.11} F_1^2 + C_{i.22} F_2^2 + \cdots\cdots + C_{i.nn} F_n^2 \\ & + C_{i.12} F_1 F_2 + \cdots\cdot + C_{i.(n-1)n} F_{(n-1)} F_n \end{aligned} \tag{2}$$

For each component, the coefficients in Equation (2) can be classified as follows:

(a) 'linear', e.g. $C_{i.j}$ for $j = 1, n$

(b) 'load squared', e.g. $C_{i.jj}$ for $j = 1, n$

(c) 'load cross product', e.g. $C_{i.jk}$ for $j = 1, n$ and $k = (j+1), n$

The number of coefficients represented by each classification is seen to be [n], [n], and [n(n-1)/2] respectively, resulting in a total number of coefficients for each component of [n(n+3)/2]. Thus, for a second order representation of a balance designed to measure six components of load and moment, the equation for each component strain gauge bridge reading contains 27 constant coefficients which must be determined by calibration. Frequently however, the coefficients in Equation (2) are assumed to depend upon the signs of the various component loads, to account for any discontinuity in the variation of the output with load occurring at zero load. This doubles the number of 'linear' and 'load squared' coefficients and quadruples the number of second order 'load cross product' coefficients; the total number of **possible** coefficients for each component then becomes [2n(n+1)] for an 'n' component balance — 84 for a 6-component balance, of which 27 define the component output for a given distribution of signs among the six components.

The objective in calibrating the balance is the determination of as many of the sensitivity and interaction coefficients as are necessary to achieve the required measurement accuracy.

2.1 Calibration by Independent Loading of Components

The most commonly used technique for calibration[3,4] of internal strain gauge balances relies upon the capability of applying load to each component independently. Thus, when a single component is loaded the output from that component is, by definition, free of any interaction from the remaining component loads which are all zero, and the outputs from the unloaded components are, again by definition, the interactions caused by the load applied to the single loaded component. The determination of the 'linear' and 'load squared' calibration coefficients by this method is thus very straightforward.

The determination of the 'load cross product' coefficients in a second order calibration requires additional 'double component' loadings where, with the procedure commonly used, the load on one component is held constant at some finite value while that on the other component is varied. Data are acquired for different values of the constant load, from which the 'cross product' coefficient can be obtained.

Consider firstly the case where only one balance component, F_j say, is loaded. The output from the strain gauge bridge for the 'i'th. component will then be given by:

$$R_i = C_{i.j} F_j + C_{i.jj} F_j^2 \tag{3}$$

Differentiation w.r.t. the variable load F_j yields:

$$\partial R_i/\partial F_j = C_{i.j} + 2 C_{i.jj} F_j \tag{4}$$

from which it is seen that the 'linear' coefficient ($C_{i.j}$) is obtained as:

$$C_{i.j} = (\partial R_i/\partial F_j)_{F_{j=0}} \tag{5}$$

Further differentiation gives:

$$\partial^2 R_i/\partial F_j^2 = 2 C_{i.jj} \tag{6}$$

from which the 'load squared' coefficient ($C_{i.jj}$) is:

$$C_{i.jj} = \frac{1}{2}(\partial^2 R_i/\partial F_j^2) \tag{7}$$

Thus, for calibrations involving the application of a single load component, the 'linear' and 'load squared' coefficients are obtained directly by fitting a second order polynomial to the variation of each component output with the applied load; the required calibration constants are the coefficients of the first and second degree terms in the applied load respectively, as indicated by Equation (3).

To determine the interaction terms involving second order cross products the two components involved must be loaded simultaneously. If this is accomplished by maintaining a constant load on component 'k' (F_k say) while varying the load F_j on component 'j', then the equation defining the output of component 'i' will be:

$$R_i = C_{i.j} F_j + C_{i.k} F_k + C_{i.jj} F_j^2 + C_{i.kk} F_k^2 + C_{i.jk} F_j F_k \tag{8}$$

and differentiation w.r.t. the variable load F_j now yields:

$$\partial R_i/\partial F_j = C_{i.j} + 2 C_{i.jj} F_j + C_{i.jk} F_k \tag{9}$$

or

$$(\partial R_i/\partial F_j)_{F_{j=0}} = C_{i.j} + C_{i.jk} F_k \tag{10}$$

The presence of the constant load F_k thus modifies the coefficient of the first degree term in the second order polynomial relating each component output to the variable load. The 'load cross product' coefficient is isolated by further differentiation w.r.t. the load F_k, requiring the acquisition of data at several different values of the constant load F_k.

Hence:

$$C_{i.jk} = \partial^2 R_i / \partial F_j \, \partial F_k \tag{11}$$

Thus, provided the balance design permits the application of calibration loads to each component independently, the procedure outlined above will allow determination of all constant coefficients in the basic balance equation — Equation (2).

2.1.1 Restrictions of the Independent Loading Method

As indicated in the Introduction, independent loading of components is not possible with some balance designs which should be classified as 'external' rather than 'internal'. Examples are balances designed to measure loads on 'half-models' or '2-dimensional' models, where the balance components are located outside the test section. In such cases, even were it possible, to apply load to a single component would obviously represent an impossible practical loading situation, contravening the basic principle that the load distribution generated by calibration should be representative of that anticipated in application.

With the development of the High Reynolds Number Two-Dimensional Test Section for the NAE 5 ft. × 5 ft. Wind Tunnel, and the associated sidewall balance system, an alternative approach was required for calibration of this balance in which no element could be loaded independently of all others. The approach developed is completely general, the case of independently loaded components described above being a special case.

2.2 A Generalized Calibration Procedure

The basic premise in the procedure to be described is that the calibration consists of applying a single varying load to the balance, in contrast to the method described above in which simultaneous application of multiple loads was necessary to obtain the non-linear load cross product terms. The location and direction of this single load defines the distribution of the applied load among the components. Whether this total (resultant) varying load results in load being applied to one or all of the balance components is immaterial, although obviously more than one component must be loaded if the non-linear load cross product terms are to be evaluated. Provided data are obtained for a sufficient number of independent loading cases, substitution of these data into the general balance equation will enable solution for the calibration coefficients.

The strain gauge bridge output for the 'i'th. component is that given by Equation (2), restated here for convenience:

$$\begin{aligned} R_i = {} & C_{i.1} F_1 + C_{i.2} F_2 + \cdots\cdots\cdots + C_{i.n} F_n \\ & + C_{i.11} F_1^{\,2} + C_{i.22} F_2^{\,2} + \cdots\cdots\cdots + C_{i.nn} F_n^{\,2} \\ & + C_{i.12} F_1 F_2 + \cdots\cdots + C_{i.(n-1)n} F_{(n-1)} F_n \end{aligned} \tag{12}$$

The output (R_i) can also be expressed as a second order polynomial in the resultant varying applied load (P) thus:

$$R_i = r_{oi} + a_i P + b_i P^2 \tag{13}$$

Differentation of (12) w.r.t. the load P yields:

$$\partial R_i/\partial P = C_{i.1}\frac{\partial F_1}{\partial P} + C_{i.2}\frac{\partial F_2}{\partial P} + \cdots\cdots + 2\,C_{i.11}\,F_1\frac{\partial F_1}{\partial P}$$

$$+ 2\,C_{i.22}\,F_2\frac{\partial F_2}{\partial P} + \cdots\cdots + C_{i.12}\left[F_1\frac{\partial F_2}{\partial P} + F_2\frac{\partial F_1}{\partial P}\right]$$

$$+ C_{i.(n-1)n}\left[F_{(n-1)}\frac{\partial F_n}{\partial P} + F_n\frac{\partial F_{(n-1)}}{\partial P}\right] \tag{14}$$

Thus,

$$(\partial R_i/\partial P)_{P=0} = a_i = C_{i.1}\frac{\partial F_1}{\partial P} + C_{i.2}\frac{\partial F_2}{\partial P} + \cdots\cdots + C_{i.n}\frac{\partial F_n}{\partial P} \tag{15}$$

and for a fixed location of the applied load we may write:

$$f_j = \frac{\partial F_j}{\partial P} = \frac{F_j}{P}$$

representing the fraction of the applied load P which is reacted by the 'j'th. balance component. For a fixed position of the applied load these 'component load distributions' or 'load ratios' are constant and defined by the static equilibrium of the system.

Equation (15) therefore becomes:

$$a_i = C_{i.1}\,f_1 + C_{i.2}\,f_2 + \cdots\cdots + C_{i.n}\,f_n \tag{16}$$

and, considering all balance component outputs, there are 'n' equations of this type each containing 'n' unknown coefficients. By acquiring data for a minimum of 'n' independent loading cases, (locations of the applied load), the resulting linear simultaneous equations can be solved to provide the linear calibration coefficients.

To define the non-linear coefficients, further differentiation of (14) w.r.t. P gives:

$$\partial^2 R_i/\partial P^2 = 2\,C_{i.11}\left(\frac{\partial F_1}{\partial P}\right)^2 + 2\,C_{i.22}\left(\frac{\partial F_2}{\partial P}\right)^2 + \cdots\cdots$$

$$+ 2\,C_{i.12}\left(\frac{\partial F_1}{\partial P}\cdot\frac{\partial F_2}{\partial P}\right) + \cdots\cdots + 2\,C_{i.(n-1)n}\left(\frac{\partial F_{(n-1)}}{\partial P}\cdot\frac{\partial F_n}{\partial P}\right)$$

or

$$\frac{1}{2}\,\partial^2 R_i/\partial P^2 = b_i = C_{i.11}\,f_1{}^2 + C_{i.22}\,f_2{}^2 + \cdots\cdots$$

$$+ C_{i.12}\,f_1\,f_2 + \cdots\cdots + C_{i.(n-1)n}\,f_{(n-1)}\,f_n \tag{17}$$

Again, the definition of a sufficient number of independent equations, [n(n+1)/2] in this case, by the application of the load P at various locations and orientations, will yield sets of linear simultaneous equations which can be solved to provide the 'load squared' and 'load cross product' calibration constants.

2.2.1 Matrix Formulation of the Equations

The sets of linear simultaneous equations defined by Equations (16) and (17) are most conveniently represented in matrix notation,

$$\underset{(n,m)}{[S]} = \underset{(n,m)}{[C]} \underset{(m,m)}{[L]} \tag{18}$$

where

- n is the number of balance components,
- m is the number of calibration coefficients per component, [m = n(n+3)/2]
- [S] contains a combination of the curve fit coefficients for each loading case, 'n' of the first degree and [n(n+1)/2] of the second degree in the applied load
- [C] contains the calibration coefficients, and
- [L] defines the distribution of the applied load P between the balance components for each loading case.

For simplicity in what follows, the number of balance components can be taken as three without loss of generality. In referring to the curve fit coefficients and the component load ratios, the first subscript will serve to define the component and the second the data for a particular loading case. For the balance calibration coefficients the subscripts refer to the 'interacted-upon' and 'interacting' components respectively in the normal way.

For the case of the six independent loading cases necessary to determine all calibration coefficients for a three-component balance, the elements of the three matrices [S], [C], and [L] are:

$$\underset{(3,9)}{[S]} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & b_{11} & b_{12} & b_{13} & b_{14} & b_{15} & b_{16} \\ a_{21} & a_{22} & a_{23} & b_{21} & b_{22} & b_{23} & b_{24} & b_{25} & b_{26} \\ a_{31} & a_{32} & a_{33} & b_{31} & b_{32} & b_{33} & b_{34} & b_{35} & b_{36} \end{bmatrix}$$

$$\underset{(3,9)}{[C]} = \begin{bmatrix} C_{1.1} & C_{1.2} & C_{1.3} & C_{1.11} & C_{1.22} & C_{1.33} & C_{1.12} & C_{1.13} & C_{1.23} \\ C_{2.1} & C_{2.2} & C_{2.3} & C_{2.11} & C_{2.22} & C_{2.33} & C_{2.12} & C_{2.13} & C_{2.23} \\ C_{3.1} & C_{3.2} & C_{3.3} & C_{3.11} & C_{3.22} & C_{3.33} & C_{3.12} & C_{3.13} & C_{3.23} \end{bmatrix}$$

$$\underset{(9,9)}{[L]} = \begin{bmatrix} f_{11} & f_{12} & f_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ f_{21} & f_{22} & f_{23} & 0 & 0 & 0 & 0 & 0 & 0 \\ f_{31} & f_{32} & f_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & f_{11}^2 & f_{12}^2 & f_{13}^2 & f_{14}^2 & f_{15}^2 & f_{16}^2 \\ 0 & 0 & 0 & f_{21}^2 & f_{22}^2 & f_{23}^2 & f_{24}^2 & f_{25}^2 & f_{26}^2 \\ 0 & 0 & 0 & f_{31}^2 & f_{32}^2 & f_{33}^2 & f_{34}^2 & f_{35}^2 & f_{36}^2 \\ 0 & 0 & 0 & f_{11} f_{21} & f_{12} f_{22} & f_{13} f_{23} & f_{14} f_{24} & f_{15} f_{25} & f_{16} f_{26} \\ 0 & 0 & 0 & f_{11} f_{31} & f_{12} f_{32} & f_{13} f_{33} & f_{14} f_{34} & f_{15} f_{35} & f_{16} f_{36} \\ 0 & 0 & 0 & f_{21} f_{31} & f_{22} f_{32} & f_{23} f_{33} & f_{24} f_{34} & f_{25} f_{35} & f_{26} f_{36} \end{bmatrix}$$

Equation (18) can be re-written, partitioning the matrices thus:

$$\underset{(3,3)}{[\ a} \ \Big| \ \underset{(3,6)}{b\ ]} = \underset{(3,3)}{[\ C1} \ \Big| \ \underset{(3,6)}{C2\ ]} \left[\begin{array}{c|c} \underset{(3,3)}{L1} & 0 \\ \hline 0 & \underset{(6,6)}{L2} \end{array}\right] \tag{19}$$

where the various sub-matrices contain:

- a — polynomial linear coefficients (a_{iz})
- b — polynomial second order coefficients (b_{iz})
- [C1] — linear calibration coefficients ($C_{i.j}$)
- [C2] — non-linear calibration coefficients ($C_{i.jj}$ & $C_{i.jk}$)
- [L1] — 'load ratios', i.e. the component loads expressed as ratios of the applied load
- [L2] — 'squares' and 'cross products' of the load ratios and subscript 'z' designates the particular loading case.

To solve for the balance calibration coefficients, Equation (19) is re-written as:

$$[C1 \,|\, C2] = [a \,|\, b] \left[\begin{array}{c|c} L1 & 0 \\ \hline 0 & L2 \end{array}\right]^{-1} \tag{20}$$

$$= [a \,|\, b] \left[\begin{array}{c|c} L1^{-1} & 0 \\ \hline 0 & L2^{-1} \end{array}\right] \tag{21}$$

from which

$$[C1] = [a] \ [L1]^{-1} \tag{22}$$

$$[C2] = [b] \ [L2]^{-1} \tag{23}$$

and clearly the solutions for the linear and second order coefficients are independent. Provided that the load ratio matrices [L1] and [L2] are non-singular, the calibration matrices are easily determined.

2.2.2 Components Loaded Independently

It is readily shown that a calibration performed by loading the balance components independently and singly is a particular case of this general solution. For this case we have:

$$F_{ii}/P = f_{ii} = 1$$

and

$$F_{ij}/P = f_{ij} = 0$$

Again using the three-component system for ease of illustration, we have:

$$[C1] = [a]\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}^{-1} = [a] \qquad (24)$$

indicating that the linear calibration coefficients are simply the coefficients of the first degree terms of the polynomial relating component output and applied load.

For the non-linear coefficients only the 'load squared' terms can be derived from single component loadings, and the matrix [L2] is consequently reduced in size from (6,6) to (3,3). Thus:

$$[C2] = [b]\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}^{-1} = [b] \qquad (25)$$

indicating that the calibration coefficients for the 'load squared' terms are simply the coefficients of the second degree terms of the load/output polynomial equation.

Equations (24) and (25) simply express the relationships:

$$C_{i.j} = (\partial R_i/\partial F_j)_{F_j = 0}$$

and

$$C_{i.jj} = \frac{1}{2}\,\partial^2 R_i/\partial F_j{}^2$$

for 'i' and 'j' varying from 1 to the number of components.

2.2.3 Effect of Constant Load on One or More Components

The procedure described in Section 2.1 utilized the technique of applying a constant load to one component in combination with a variable load on another in order to determine the coefficient of the second order load cross product in question. However, the formulation just described for calibration using multi-component loading, is based on the premise that the only load applied to the balance is a single varying one and that consequently each of the individual component loads is proportional to this load. This will no longer be true if a constant load is applied to one or more components, in addition to the variable load. The formulation is extended below to allow the use of this calibration procedure.

Assume again an 'n' component balance having a second order load/output relationship, and let the calibration procedure consist of applying a single resultant varying load (P) in combination with one or more constant component loads (K_i, $i <= n$).

In terms of the balance calibration coefficients, the output of the 'i'th. component is now:

$$R_i = C_{i.1}(F_1 + K_1) + C_{i.2}(F_2 + K_2) + \cdots\cdots + C_{i.n}(F_n + K_n)$$
$$+ C_{i.11}(F_1 + K_1)^2 + C_{i.22}(F_2 + K_2)^2 + \cdots\cdots + C_{i.nn}(F_n + K_n)^2$$
$$+ C_{i.12}(F_1 + K_1)(F_2 + K_2) + C_{i.13}(F_1 + K_1)(F_3 + K_3) + \cdots$$
$$+ C_{i.(n-1)n}(F_{(n-1)} + K_{(n-1)})(F_n + K_n) \qquad (26)$$

and in terms of the variable applied load (P) it is, as before:

$$R_i = r_{oi} + a_i P + b_i P^2$$

Differentiating Equation (26) w.r.t. the variable load (P) yields:

$$\partial R_i/\partial P = C_{i.1} f_1 + C_{i.2} f_2 + \cdots\cdots + C_{i.n} f_n$$
$$+ 2 C_{i.11}(F_1 + K_1) f_1 + \cdots\cdots + 2 C_{i.nn}(F_n + K_n) f_n$$
$$+ C_{i.12}[(F_1 + K_1) f_2 + (F_2 + K_2) f_1] + \cdots\cdots$$
$$+ C_{i.(n-1)n}[(F_{(n-1)} + K_{(n-1)}) f_n + (F_n + K_n) f_{(n-1)}] \qquad (27)$$

and hence

$$a_i = \left.\frac{\partial R_i}{\partial P}\right|_{P=0} = C_{i.1} f_1 + \cdots + C_{i.n} f_n + 2 C_{i.11} f_1 K_1 + \cdots$$
$$+ 2 C_{i.nn} f_n K_n + C_{i.12}(f_1 K_2 + f_2 K_1) + \cdots$$
$$+ C_{i.(n-1)n}(f_{(n-1)} K_n + f_n K_{(n-1)}) \qquad (28)$$

Further differentiation w.r.t. P gives:

$$b_i = \frac{1}{2}\frac{\partial^2 R_i}{\partial P^2} = C_{i.11} f_1{}^2 + \cdots\cdots + C_{i.nn} f_n{}^2$$
$$+ C_{i.12} f_1 f_2 + \cdots + C_{i.(n-1)n} f_{(n-1)} f_n \qquad (29)$$

It can be seen that the presence of the constant loads affects only the coefficient of the first degree term in the load/output equation, and now it is possible, in principle, to solve for all calibration constants, linear and non-linear, using only the 'a' coefficients from the curve fitting of the calibration data.

In matrix notation:

$$\underset{(n,m)}{[a]} = \underset{(n,m)}{[C1 \;\vdots\; C2]} \; \underset{(m,m)}{[L^*]} \tag{30}$$

where 'm' is the total number of calibration coefficients per component, and the elements in a column of the matrix [L*] are functions of the 'load ratios' and the constant loads as indicated;

$$\{f_1, f_2, f_3, \text{----} f_n,$$

$$2\,f_1\,K_1, 2\,f_2\,K_2, 2\,f_3\,K_3, \text{----} 2\,f_n\,K_n,$$

$$[f_1\,K_2 + f_2\,K_1], \text{----} [f_{(n-1)}\,K_n + f_n\,K_{(n-1)}]\}$$

Using the 'b' polynomial coefficients we have, as previously,

$$\underset{(n,p)}{[b]} = \underset{(n,p)}{[C2]} \; \underset{(p,p)}{[L2]} \tag{31}$$

where 'p' is the number of non-linear calibration coefficients per component.

Whilst in principle Equation (30) can be used to determine all coefficients, to do so requires the acquisition of data for [n(n+3)/2] independent loading situations. This requirement can be reduced by [n] cases if Equation (31) is used to solve for the second order coefficients, (matrix [C2]), as in this case only [n(n+1)/2] load applications are required. By writing Equation (30) in the form:

$$\underset{(n,n)}{[a]} = \underset{(n,m)}{[C1 \;\vdots\; C2]} \; \underset{(m,n)}{\begin{bmatrix} L1^* \\ \hline L2^* \end{bmatrix}}$$

$$= \underset{(n,n)}{[C1]} \; \underset{(n,n)}{[L1^*]} + \underset{(n,p)}{[C2]} \; \underset{(p,n)}{[L2^*]} \tag{32}$$

where $p = (m - n)$,

we see that by substituting the known matrix [C2], the 'a' values can be corrected for the effect of the constant loads. The linear calibration coefficients are then obtainable from the corrected 'a' values thus:

$$[C1] = \{[a] - [C2]\,[L2^*]\}\,[L1^*]^{-1} \tag{33}$$

Solution of this equation requires only 'n' loading cases, which may be chosen from the [n(n+1)/2] already used to solve for the second order constants.

If, as a result of the loading procedure used, the matrix [L2] in Equation (31) should be singular, it becomes necessary to solve for a matrix [C2] containing only the 'load squared' calibration coefficients. This situation will result if the procedure described in Reference 3 and Section 2.1 is followed, whereby the balance components are loaded independently. In this case the product of the 'load ratios' of any two components would always be zero, the 'load ratios' being the ratios of the single variable applied load reacted by each component, and therefore unaffected by the presence of constant loads. Removal of the load ratio product terms from the matrix [L2] would allow determination of the 'load squared' calibration coefficients by substitution of 'n' independent loading cases into Equation (31). The procedure for correcting the 'a' values is still applicable, but the matrix [C1] in Equation (33) would now be composed of both the 'linear' and the 'load cross product' calibration coefficients, and [n(n+1)/2] load cases would be required to achieve the solution. The matrices [L1*] and [L2*] would, of course, be modified to reflect the changed composition of [C1] and [C2].

2.2.4 Advantages of the Generalized Procedure

The most obvious advantage is that it is equally applicable to either 'internal' or 'external' types of balances. Also the formulation is such that the calibration constants determined by the procedure relate to the 'fundamental' components and not to 'derived' components. Here 'fundamental' components are defined as those into which the system of elastic hinges and links resolves the applied load and moment; 'derived' components are those obtainable from combinations of these fundamental reactions. For example, a simple two-component balance might have, as fundamental components, the forces reacted at two locations in the same plane. These two forces also define a resultant force and a moment but, according to the definition used here, these are derived rather than fundamental components. The importance of maintaining this distinction becomes apparent when considering the use of separate calibrations for positive and negative loads in each component. As mentioned before, this is frequently done to compensate for discontinuities in the load/output relationship at zero load, however such a distinction is meaningful only when considered in terms of the fundamental components. For example, in the case of the two-component balance just cited, a positive total resultant load (a derived component) indicates only that both fundamental component loads cannot be negative, and likewise a particular sign of the total moment may result from three of the four possible sign combinations of the fundamental components. Clearly the behaviour of the balance depends on the signs of the reactions in the basic flexure elements and not on the signs of the total resultant force and moment.

The concept of having multiple components loaded simultaneously allows the calibration procedure to be much more representative of the manner in which the balance will be used in practice. From a purely theoretical point of view, provided that the actual behaviour is exactly as defined by the chosen equation for the load/output relationship, the precise combination of loading arrangements used in calibration is immaterial; all would yield exactly the same result. However, in practice the chosen polynomial relationship, whether it be limited to second order or includes higher order terms, will not describe exactly the behaviour of each component, and this results in the solution of the system of equations being dependent upon the conditions represented by the equations. In other words, the calibration coefficients will depend upon the manner in which the balance was loaded during calibration, as different loading distributions will result in different deflection characteristics. This being so, there would appear to be a strong argument in favour of applying, during calibration, loads which generate distributions representative of those likely to be experienced in use of the balance to measure model forces in a wind tunnel. The occurrence of single component loadings in a practical test situation would be very infrequent, and accordingly the preferred method of calibration would seem to be the proposed multi-component loading procedure.

This procedure may also enable a satisfactory accuracy to be obtained without the need to make a complete non-linear calibration, something which can realize a considerable saving in calibration effort. This comes about by virtue of the fact that, during a calibration involving the loading of several components simultaneously, the strain gauge bridge outputs automatically include the effects, linear and non-linear, of all components having non-zero loads. If these outputs are now 'linearized' by straight line curve fitting of the output data with respect to the applied load, the 'slopes' will take into account, in a linear fashion, all of the 'load squared' and 'load cross product' non-linear effects of those components with non-zero load. In the case of a single component loading only the 'load squared' effect of the loaded element would be present. Provided that the non-linear effects are reasonably small, it is possible that a linear calibration utilizing compound loading of the components may provide adequate accuracy, better than that obtainable from a linear calibration determined from single component loadings.

2.2.5 An Optimized Solution of the Calibration Equations

As the calibration coefficients will depend upon the particular set of equations solved (for a practical situation), and as the formulation permits the use of calibration loadings chosen to be representative of the expected test measurement environment, this procedure lends itself to the idea of obtaining an optimum solution from a large number of calibration loadings. These could be made to cover, as completely as possible, the expected magnitude and location of the test load, and the resulting overdetermined system of equations could then be solved, in a least squares fashion[5], to provide the optimum solution.

Assume that we wish the solution to an overdetermined system of equations, of the form of Equation (30), for the (n,m) matrix of calibration coefficients for an 'n'-component balance, utilizing 'z' loading cases where 'z' is greater than 'm', and 'm'=[n(n+3)/2].

We therefore require to solve for the linear [C1] and non-linear [C2] calibration matrices in:

$$\underset{(n,z)}{[a]} = \underset{(n,m)}{[C1 \;\vdots\; C2]} \; \underset{(m,z)}{[L^*]} \tag{34}$$

This overdetermined system of equations can be solved, in a least squares sense, by post-multiplication of each side of Equation (34) by the transpose of matrix [L*]. Thus:

$$\underset{(n,z)}{[a]} \; \underset{(z,m)}{[L^*]^T} = \underset{(n,m)}{[C1 \;\vdots\; C2]} \; \underset{(m,z)}{[L^*]} \; \underset{(z,m)}{[L^*]^T} \tag{35}$$

and hence

$$\underset{(n,m)}{[C1 \;\vdots\; C2]} = \underset{(n,z)}{[a]} \; \underset{(z,m)}{[L^*]^T} \underset{(m,m)}{\left[\underset{(m,z)}{[L^*]} \; \underset{(z,m)}{[L^*]^T} \right]^{-1}} \tag{36}$$

In this manner many different loading distributions may be permitted to influence the resulting calibration coefficients, which represent a least squares solution of all the equations (load cases) involved; weighting factors may be applied to particular loading configurations if desired.

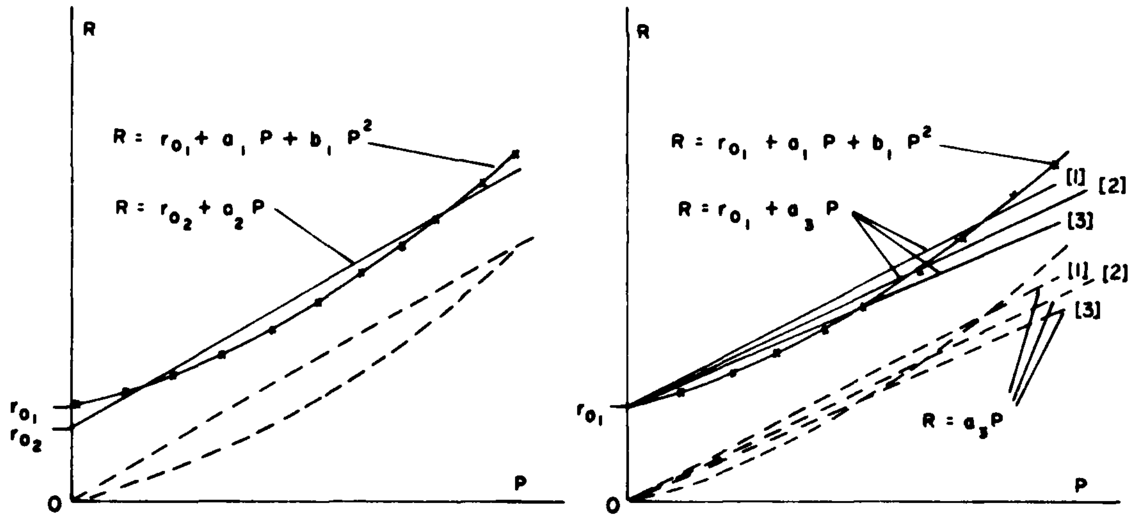
2.3 Some Notes on 'Zeros' and Intercepts in Curve Fitting

Fundamental to all methods of reducing calibration data to the form of sensitivity and interaction constants, is the process of obtaining the coefficients in the polynomial load/output equation which provide a best fit to the experimental data. In the foregoing, the equation chosen has been a second order polynomial for the general case, although obviously a straight line fit would be used if a linear calibration was the objective.

The presence of other than first degree terms presents the problem of defining what the true 'zero' on the load scale should be as, for anything other than a linear relationship, the use of one 'zero' for calibration and another for application will give rise to errors. As indicated in the Introduction, this report will describe a procedure for the determination of 'buoyant' (i.e. zero absolute load) component 'zeros' for reduction of data from wind tunnel balances. Accordingly, if the same 'buoyant' condition is used as the 'zero' during calibration of the balance the potential problem of initial load effects[6,7] is avoided. With the extended formulation given in Section 2.2.3 this objective is easily achieved by specifying the calibration tare loads, (i.e. the weight of the calibration body and the 'metric mass' of the balance itself), as constant loads existing during the calibration. The precise distribution of this tare weight between the balance components will depend upon the balance orientation and the centre of gravity of the tare mass, and must be determined for each loading case. Of course, the significance of the tare load lies in its magnitude relative to the load range over which the balance is to be used, and also in the relative importance of non-linear effects for the balance; a consideration of these facts will indicate whether or not the tare load need be considered for a particular circumstance.

The preceeding discussion of 'zeros' was concerned with zeros on the load scale. The zeros for the strain gauge bridge outputs are the intercepts (r_0) in Equation (13), and are of no real significance as balance data reduction is concerned with incremental changes and the intercept values cancel in the process of tare subtraction. The main significance of the intercept is that, in general, the curve fitting should be performed with a 'free' intercept in order to obtain the best fit to the experimental data. Having found the coefficients in Equation (13) — 'r_0', 'a', and 'b' — which define this best fit, the intercept values may be disregarded.

There is one exception to this general rule, namely the fitting of a straight line to data which contains some non-linear effects. An example would be in making a linear analysis of calibration data from multi-component loadings, as suggested in Section 2.2.4.



Suppose that the experimental output is well fitted by the second order polynomial:

$$R = r_{o1} + a_1 P + b_1 P^2$$

as indicated in the diagrams above. If this same data is fitted, using a least squares technique, by the straight line:

$$R = r_{o2} + a_2 P$$

the result will be that the sum of the deviations of the data on either side of the line will cancel. The greatest deviations will therefore occur at the end points and in the middle of the range as shown in the left hand diagram.

Now consider both of these curves translated along the 'output' axis so as to pass through the origin, as indicated by the dashed lines in the diagram — this is the equivalent of subtracting a tare value. We see that the straight line now gives a maximum deviation from the data at the middle of the range, and that all points on it are biased in one direction relative to the experimental data. This is clearly an undesirable situation, but it can be corrected by forcing the linear fit to pass through the same origin as the second order fit; i.e. by fitting the data to the straight line:

$$R = r_{o1} + a_3 P$$

In practice this is accomplished by subtracting the value r_{o1} from all experimental outputs and then fitting with the straight line:

$$R = a_3 P$$

For a least squares fit of this equation we must minimize the sum of the squares of the residuals w.r.t. the coefficient 'a_3'; i.e.

$$\Sigma\, [R_i - a_3 P_i]^2$$

Differentiating w.r.t. 'a_3' and equating to zero:

$$2\, \Sigma\, P_i\, [R_i - a_3 P_i] = 0$$

indicating that the deviations from the straight line, when weighted by P_i, will sum to zero; this will bias the line toward data at large values of P_i which is undesirable — line [1] in the right hand diagram.

To obtain a better fit, in which the deviations will simply sum to zero, it is necessary to weight each point by a factor inversely proportional to the square root of the load P_i. Thus we require to minimize:

$$\Sigma \left[\frac{1}{P_i^{1/2}} (R_i - a_3 P_i) \right]^2$$

Differentiating w.r.t. 'a_3' and equating to zero:

$$2\, \Sigma\, [R_i - a_3 P_i] = 0$$

and the required slope of the straight line is seen to be simply:

$$a_3 = \Sigma\, R_i / \Sigma\, P_i$$

This straight line is such that the deviations of the data on either side of the line cancel — a common interpretation of 'best fit'. A comparison of this, (line [2]), and the original data is given in the right hand diagram; it will be noted that the uni-directional bias no longer exists, and that the deviation generally increases as the load increases. This trend is quite acceptable as greater absolute errors can be tolerated when the load is large.

Following a recent discussion* on this topic it was suggested to the author that a logical extension to the approach of balancing the deviations on either side of the fitted straight line would be to balance the deviations **when expressed as fractions of the local data value.** This would come closest to the 'ideal' of error being a constant percentage of measurement.

To achieve such a 'fit' each point must be weighted by the factor:

$$1/(R_i\,P_i)^{1/2}$$

and we require to minimize

$$\Sigma \left[\frac{1}{(R_i\,P_i)^{1/2}} (R_i - a_3\,P_i) \right]^2$$

Differentiating w.r.t. 'a_3' and equating to zero:

$$2\,\Sigma \left[1 - a_3 \left(\frac{P_i}{R_i} \right) \right] = 0$$

from which the coefficient a_3 is seen to be:

$$a_3 = N/\Sigma \left(\frac{P_i}{R_i} \right)$$

for N data values. The straight line for which the slope (a_3) is defined in this way, (line [3]), would seem to be preferable to the one for which the unweighted deviations sum to zero.

Thus, contrary to what might be anticipated, in making a linear curve fit of experimental balance calibration data, the best technique is to force the straight line to pass through the experimental output value at zero load, or alternatively, through the intercept value obtained from a second order curve fit of the data. This will eliminate the uni-directional bias resulting from the use of a 'free' intercept in the curve fit, combined with the subtraction of 'tares' in reducing the data which results in cancellation of the common intercept value. In addition, suitable weighting of the data can be used to achieve a balance of the deviations, expressed as percentage of measurement, on either side of the line.

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2.4 Normalization of the Calibration Matrix

As the constants in the linear [C1] and non-linear [C2] calibration matrices are closely related to the polynomial coefficients of the first and second degree in the load respectively, it is obvious that they are not non-dimensional. The linear constants have units of 'output unit per unit of load', while those for the non-linear constants are 'output unit per unit of load squared'. Consequently the complete matrix of calibration constants is applicable only when the balance is used with a system having the same output units as the calibration system.

Rather than having to re-specify all of the constants to suit a different output system, a 'normalized' form of the matrix can be used, in which the constants in the equation for each component are expressed as ratios of the primary sensitivity of the respective component. These primary sensitivities are the diagonal elements of the linear calibration matrix [C1], and, following the normalization procedure, are the only elements of the calibration matrix which retain a dependence on the system used to measure the strain gauge bridge outputs. A factor for converting from one output system to another can be obtained readily by either check loading individual components, (so as to exclude interactions), or by imbalancing the individual component strain gauge bridges using shunt resistors. The primary sensitivities are multiplied (or divided), by this factor to convert them from one system to the other.

Normalization is achieved by division of the constants in the equation for each component by the primary sensitivity of that component. This corresponds to pre-multiplication of the original calibration matrix by a diagonal matrix [D] composed of the reciprocals of the diagonal elements of the linear matrix [C1]. The resulting 'interaction' matrix [X] is then:

$$\underset{(n,m)}{[X]} = [\underset{(n,n)}{X1} \;\vdots\; \underset{(n,p)}{X2}] = \underset{(n,n)}{[D]}\;[\underset{(n,n)}{C1} \;\vdots\; \underset{(n,p)}{C2}] \tag{37}$$

where the elements of [X] have universal application, but those of [D] are dependent on the particular readout system.

The ultimate check of the complete calibration procedure can now be made by applying check loads to the balance and using the derived calibration coefficients to re-calculate the loads from the measured component outputs. Several procedures for solution of the balance equations will now be compared.

3.0 REDUCTION OF STRAIN GAUGE BALANCE DATA

When the balance components are loaded, each component strain gauge bridge exhibits a reading, related to all of the components acting, which is defined by the equation:

$$\underline{R} = [C1]\,\underline{F} + [C2]\,\underline{F}^* \tag{38}$$

where

- $\underline{R}$ is the vector of outputs at unit excitation voltage,
- [C1] is the matrix of linear calibration coefficients,
- [C2] is the matrix of non-linear calibration coefficients,
- $\underline{F}$ is the vector of component loads,

and

- $\underline{F}^*$ is a vector composed of the squares and cross-products of the component loads.

A common procedure for determination of the force vector $\underline{F}$ in this equation is given in Reference 3. This form of solution, which requires iteration, is restated here as it is the correct procedure fundamentally. It is compared with an approximate solution procedure known to have been used, and perhaps still in use, at some wind tunnel test facilities in Europe. This latter approach has in its favour the fact that the approximation assumption avoids the need for an iterative type of solution; whether this simplification justifies the approximation involved is another question.

Pre-multiplication of both sides of Equation (38) by the diagonal matrix [D] in Equation (37) gives:

$$[D]\,\underline{R} = [D]\,[C1]\,\underline{F} + [D]\,[C2]\,\underline{F}^*$$

or

$$\underline{F}^{\sim} = [X1]\,\underline{F} + [X2]\,\underline{F}^* \tag{39}$$

where the vector $\underline{F}^{\sim}$ consists of the approximate component loads (ignoring interactions) resulting from division of each component output by the appropriate primary sensitivity constant.

Solving for the force vector $\underline{F}$ gives:

$$\underline{F} = [X1]^{-1}\,\underline{F}^{\sim} - [X1]^{-1}\,[X2]\,\underline{F}^* \tag{40}$$

For balances having an assumed linear behaviour the solution is given explicitly by:

$$\underline{F} = [X1]^{-1}\,\underline{F}^{\sim} \tag{41}$$

For non-linear behaviour the iterative solution is commenced using the solution of Equation (41), $\underline{F}_1$ say, to calculate a first approximation of the non-linear term ($\underline{\Delta}_1$) thus:

$$\underline{\Delta}_1 = [X1]^{-1}\,[X2]\,\underline{F}_1^* \tag{42}$$

where the elements of the vector $\underline{F}_1^*$ are composed of the squares and cross-products of the elements of the approximate vector $\underline{F}_1$.

A second approximation ($\underline{F}_2$) is then obtained as:

$$\underline{F}_2 = \underline{F}_1 - \underline{\Delta}_1 \tag{43}$$

from which $\underline{F}_2^*$ may be determined and then substituted into Equation (42) to provide a further estimate of the non-linear term.

In general the final solution is given by:

$$\underline{F}_n = \underline{F}_1 - \underline{\Delta}_{(n-1)} \tag{44}$$

where 'n' is the number of iterations.

Any iterative solution raises questions as to whether or not successive approximations will always converge, and, if they do, whether the limit is a unique solution of the given equations. These questions are considered in Reference 6 where it is shown that a converged solution represents a unique solution, and that the question of whether or not convergence takes place is dependent upon the balance interaction coefficients and the design loads only. A criterion is given which can be evaluated for a given balance prior to its use; for typical balances the non-linear effects are generally quite small and convergence is usually rapid.

3.1 An 'Explicit' Approximate Solution

In Equation (40) the elements of the non-linear force vector $\underline{F}^*$ are functions of the squares and cross-products of the true load vector $\underline{F}$, i.e.

$$\underline{F}^* = f(\underline{F})$$

In the method to be outlined it is assumed that the non-linear terms are sufficiently small, in comparison with the linear terms, that the non-linear vector $\underline{F}^*$ may be approximated using the elements of the vector $\underline{F}_1$ obtained by ignoring non-linear effects. Thus

$$\underline{F}^* \simeq \underline{F}^{\sim *} = f(\underline{F}_1)$$

where

$$\underline{F}_1 = [X1]^{-1}\, \underline{F}^{\sim} \tag{45}$$

Conceding for the moment the validity of this approximation, it is now possible to substitute for $\underline{F}^*$, (in terms of $\underline{F}^{\sim}$), in Equation (40), thus making the solution for the true load vector ($\underline{F}$) explicit in terms of $\underline{F}^{\sim}$ and the non-linear vector $\underline{F}^{\sim *}$ where:

$$\underline{F}^{\sim *} = f(\underline{F}^{\sim}) \quad \text{and} \quad \underline{F}^{\sim} = [D]\, \underline{R}$$

as seen from Equation (39).

References 8 and 9 describe analytically ways in which this substitution for the non-linear vector can be made. However, the following computational method is offered here as it provides a means of 'tailoring' the assumption to particular load ranges and distributions.

It is required to determine the coefficient matrices in the expression:

$$\underline{F} = [X1]^{-1}\, \underline{F}^{\sim} - [B]\, \underline{F}^{\sim *} \tag{46}$$

Restating Equation (40) we have:

$$\underline{F} = [X1]^{-1}\, \underline{F}^{\sim} - [X1]^{-1}\, [X2]\, \underline{F}^*$$

and from a comparison of this with Equation (46) it is clear that:

$$[B]\, \underline{F}^{\sim *} = [X1]^{-1}\, [X2]\, \underline{F}^* \tag{47}$$

Now the interaction matrices [X1] and [X2] are known, and for any specified force vector $\underline{F}$ we may compute the output vector $\underline{R}$ from Equation (38) and hence $\underline{F}^{\sim}$. Then:

$$\underline{F}^{*} = f(\underline{F})$$

and

$$\underline{F}^{\sim *} = f(\underline{F}^{\sim})$$

Thus, by computing the vectors $\underline{F}^{*}$ and $\underline{F}^{\sim *}$ corresponding to a sufficient number of input load vectors $\underline{F}$, the required matrix [B] is obtained from the equation:

$$\underset{(n,p)}{[B]} = \underset{(n,n)}{[X1]^{-1}}\ \underset{(n,p)}{[X2]}\ \underset{(p,p)}{[F^{*}]}\ \underset{(p,p)}{[F^{\sim *}]^{-1}} \tag{48}$$

where p = [n(n+1)/2] and 'n' is the number of components.

The 'p' columns of [F*] and [F$^{\sim}$*] are composed of the corresponding vectors, ($\underline{F}^{*}$ and $\underline{F}^{\sim *}$), for each input load vector $\underline{F}$. Thus, by appropriate choice of these load vectors, the solution for [B] can be 'tailored' to specific combinations and ranges of the individual component loads. The value of [B] determined from the analytic treatments of References 8 and 9 is obtained when the magnitudes of the component loads in the chosen vectors are made to approach zero. As 'p' input load vectors are used in the solution for [B], it follows that these particular vectors, i.e. component load distribution and magnitudes, must be exact solutions of Equation (47), and thus of both the approximate and the iterative solution methods. Use of the approximate method to compute load conditions close to any of these particular load vectors will thus involve little error.

Use of 'q' input load vectors, (q > p), in solving for [B] results in an overdetermined set of equations which may be solved, in a least squares sense[5], by:

$$\underset{(n,p)}{[B]} = \underset{(n,n)}{[X1]^{-1}}\ \underset{(n,p)}{[X2]} \left[\underbrace{\underset{(p,q)}{[F^{*}]}\quad \underset{(q,p)}{[F^{\sim *}]^{T}}}_{(p,p)}\right] \left[\underbrace{\underset{(p,q)}{[F^{\sim *}]}\quad \underset{(q,p)}{[F^{\sim *}]^{T}}}_{(p,p)}\right]^{-1} \tag{49}$$

thus allowing a greater number of load vectors to influence the result, although none will be an exact solution of Equation (47).

This approach to balance data reduction, in addition to involving an approximation, suffers from the disadvantage that it is not amenable to the use of different positive load and negative load sensitivity and interaction coefficients; what might be termed 'plus-minus' coefficients. This is easily seen from the fact that the product of [X2] and the inverse of [X1] is involved in the determination of [B], which therefore will be valid only for that particular 'sign of load' distribution. The only advantage of the approach seems to be that no iteration is involved in the solution; given the rapid convergence of the iterative solution for typical balances, and the speed of modern computers, this does not seem to the author to be sufficient justification for an 'unnecessary' approximation. The method is not employed at NAE for reduction of balance data from the 5 ft. × 5 ft. Wind Tunnel.

3.2 Accounting for 'Plus-Minus' Calibration Coefficients

Calibrations of strain gauge balances which separate the behaviour of the fundamental components, (i.e. the flexure elements), to positive and negative loading are quite common. The inherent 'cubic' type of load/output relationship for typical bending beam flexure elements is fitted better by a second order polynomial if the positive and negative load data are assessed separately. Two methods are presented to account for this dependence of the calibration coefficients on the sign of the loads.

3.2.1 Synthesis of a Specific Matrix for Each Load Vector

The calibration coefficients defining the balance behaviour, in number and type, are:

'linear'	[2n], + and - loads
'load squared'	[2n], + and - loads
'load cross-product'	[2n(n-1)], ++, +-, -+, and -- combinations of loads.

Thus, in total, [2n(n+1)] coefficients must be stored for use by the data reduction programme.

In solving for the force vector corresponding to a given output vector only [n] 'linear', [n] 'load squared', and [n(n-1)/2] 'load cross-product' coefficients are involved for any given sign combination. Thus, if the combination of signs in the required load vector can be established, then the appropriate linear [X1] and non-linear [X2] matrices can be synthesized from the stored coefficients, and Equation (40) solved to yield the load vector. Note that the solution requires the inversion of matrix [X1] and determination of the product of this inverse and the non-linear matrix [X2].

The difficulty lies in establishing the correct sign distribution upon which to base the choice of the appropriate coefficients. Strictly these must be chosen according to the signs of the loads but these are, as yet, unknown. The only option is to select the matrix elements on the basis of the signs of the component outputs, following subtraction of tares to remove any 'offsets'. However, depending on the exact definition of the tares used, this distribution of the signs of the outputs may not correspond to the distribution of the signs of the loads. For example, subtraction of a tare corresponding to the model weight will give the indication of a zero output when actually the components which react the weight are negatively loaded. It is necessary therefore, to compare the distribution of signs in the calculated load vector with that used in selecting the matrix elements; if differences exist a new selection of the matrix elements must be made, using the updated sign distribution, and the load vector re-computed. This procedure should be repeated until the correct load distribution is found.

The use of 'buoyant' tares, to be described later, facilitates the determination of the correct sign distribution by defining the tares as the component outputs at zero absolute load. Consequently, following subtraction of these 'buoyant' tares, the sign distribution of the output vector and that of the load vector are generally identical and no iteration is required. It is possible that interactions could cause a difference between the two distributions for components which are very lightly loaded; however, this situation would likely result in failure of the iterative scheme to converge to a unique solution, providing instead a 'flip-flop' between two load distributions. Under these circumstances the choice would be arbitrary in any event.

3.2.2 Concept of Load and Absolute Value of Load

In this approach Equation (39) is rewritten to express the elements of the vectors $\underline{F}$ and $\underline{F}^*$ in terms of the 'signed' and 'absolute' values of the individual component loads. The linear term in Equation (39) thus becomes:

$$[X1]\ \underline{F} = [XL1]\ \underline{F} + [XL2]\ \underline{FL2}$$

where

[XL1] and [XL2] are (n,n) matrices appropriate to the 'signed' and 'absolute value' load vectors $\underline{F}$ and $\underline{FL2}$ respectively.

The second order term can be divided into two 'load squared' and four 'load cross-product' contributions thus:

$$[X2]\ \underline{F}^* = [XSQ1]\ \underline{FSQ1} + [XSQ2]\ \underline{FSQ2}$$
$$+ [XCP1]\ \underline{FCP1} + [XCP2]\ \underline{FCP2}$$
$$+ [XCP3]\ \underline{FCP3} + [XCP4]\ \underline{FCP4}$$

where

[XSQ1] and [XSQ2] are (n,n) matrices appropriate to the 'load squared' vectors $\underline{FSQ1}$ and $\underline{FSQ2}$ respectively,

[XCP1], [XCP2], [XCP3], and [XCP4] are (n,p) matrices appropriate to the 'load cross-product' vectors $\underline{FCP1}$, $\underline{FCP2}$, $\underline{FCP3}$, and $\underline{FCP4}$ respectively,

and

$p = [n(n-1)/2]$ as usual.

The general elements for all of the load vectors noted are as follows:

$\underline{F}$: $\{(F_i),\ i = 1, n\}$

$\underline{FL2}$: $\{(|F_i|),\ i = 1, n\}$

$\underline{FSQ1}$: $\{(F_i \cdot F_i),\ i = 1, n\}$

$\underline{FSQ2}$: $\{(F_i \cdot |F_i|),\ i = 1, n\}$

$\underline{FCP1}$: $\{[(F_i \cdot F_j),\ i = 1, n],\ j = (i+1), n\}$

$\underline{FCP2}$: $\{[(F_i \cdot |F_j|),\ i = 1, n],\ j = (i+1), n\}$

$\underline{FCP3}$: $\{[(|F_i| \cdot F_j),\ i = 1, n],\ j = (i+1), n\}$

$\underline{FCP4}$: $\{[(|F_i \cdot F_j|),\ i = 1, n],\ j = (i+1), n\}$

where

F_i and F_j are the component loads.

Using these vectors and calibration matrices, Equation (39) becomes:

$$\underline{F}^{\sim} = [XL1]\ \underline{F} + [XL2]\ \underline{FL2} + [XSQ1]\ \underline{FSQ1} + [XSQ2]\ \underline{FSQ2}$$
$$+ [XCP1]\ \underline{FCP1} + [XCP2]\ \underline{FCP2} + [XCP3]\ \underline{FCP3} + [XCP4]\ \underline{FCP4} \qquad (50)$$

and the solution for the load vector $\underline{F}$ is now given by:

$$\underline{F} = [XL1]^{-1}\ \underline{F}^{\sim} - [XL1]^{-1} \{ [XL2]\ \underline{FL2} + [XSQ1]\ \underline{FSQ1} + [XSQ2]\ \underline{FSQ2}$$
$$+ [XCP1]\ \underline{FCP1} + [XCP2]\ \underline{FCP2} + [XCP3]\ \underline{FCP3} + [XCP4]\ \underline{FCP4} \} \qquad (51)$$

This appears more complex than Equation (40) but it should be noted that the inverse of the matrix [XL1], and the product of this and the other matrices, can be pre-stored as they are independent of the sign distribution of the load vector.

Defining the general elements of the 'linear', 'load squared', and 'load cross-product' coefficient matrices as:

'linear'	— $\ell_{i.j}^{+}, \ell_{i.j}^{-}$
'load squared'	— $q_{i.j}^{+}, q_{i.j}^{-}$
'load cross-product'	— $c_{i.j}^{++}, c_{i.j}^{+-}, c_{i.j}^{-+}, c_{i.j}^{--}$

the expressions for the general elements of each of the derived matrices are:

[XL1]	: $(\ell_{i.j}^{+} + \ell_{i.j}^{-})/2$
[XL2]	: $(\ell_{i.j}^{+} - \ell_{i.j}^{-})/2$
[XSQ1]	: $(q_{i.j}^{+} + q_{i.j}^{-})/2$
[XSQ2]	: $(q_{i.j}^{+} - q_{i.j}^{-})/2$
[XCP1]	: $(c_{i.j}^{++} + c_{i.j}^{--} + c_{i.j}^{+-} + c_{i.j}^{-+})/4$
[XCP2]	: $(c_{i.j}^{++} - c_{i.j}^{--} - c_{i.j}^{+-} + c_{i.j}^{-+})/4$
[XCP3]	: $(c_{i.j}^{++} - c_{i.j}^{--} + c_{i.j}^{+-} - c_{i.j}^{-+})/4$
[XCP4]	: $(c_{i.j}^{++} + c_{i.j}^{--} - c_{i.j}^{+-} - c_{i.j}^{-+})/4$

3.2.3 Comparison of the Two Approaches

Firstly, for both approaches the storage requirement for the coefficient matrices is identical — [2n(n+1)] coefficients per component for a complete second order system.

The first method requires, for each load vector to be determined, synthesis of the calibration matrix for the appropriate distribution of load signs, and a matrix inversion and multiplication to determine:

$$[X1]^{-1} \quad \text{and} \quad [X1]^{-1}[X2]$$

in Equation (40). This procedure may have to be repeated in order to arrive at the correct sign distribution for synthesis of the matrix, unless the 'buoyant' tares are utilized. Solution for the load vector is explicit for a linear calibration, and iterative for a non-linear calibration.

The second method utilizes eight pre-stored matrices, (two 'linear', two 'load squared', and four 'load cross-product'), to eliminate the requirement to select and invert a matrix for each load vector. However, a wider range of load-related vectors must be calculated, and a larger number of matrix multiplications are required. The solution is iterative for both linear and non-linear calibrations.

Some tests were made to determine the relative computation time requirements of the two methods, by solving for a series of load vectors (for a six-component system) using:

(a) a linear calibration,

(b) a non-linear calibration, ignoring all second order 'load cross-product' terms, and

(c) a complete second order non-linear calibration.

In all cases the method involving synthesis of a new matrix for each load vector proved to be less efficient; for cases (a) and (b) the ratio of the computation times was approximately 3.5, which was reduced to approximately 2 by inclusion of the 'load cross-product' terms in case (c). Thus, if computation time is considered to be a significant factor, the method involving the use of separate pre-stored coefficient matrices would seem to be preferable.

4.0 THE APPLICATION OF FORCE BALANCE TARES

In a wind tunnel test measuring model forces, meaningful aerodynamic data is obtained only after determination and removal of the model weight effects from the measured data. The distribution of the model weight tares among the balance components is a function of the magnitude and centre of gravity position of the 'metric mass' and the balance orientation relative to a gravitational axis system. However, unless the orientation is sensed within the model/balance assembly, as a result of support deflections it will depend upon the total load acting. Thus the tare values cannot be determined without knowing the orientation, which cannot be determined without knowing the total load, which, in turn, cannot be determined without knowing the tare values, ... ; the solution thus requires iteration.

This requirement for iteration arises from the fact that the test (wind-on) measurements are regarded as being incremental from some finite-load tare condition, for which the component loads, in a balance-axes system, depend upon the orientation. If instead one supposes an 'absolute zero load' or 'buoyant' condition for the tare, (which, by definition, will be independent of the orientation, 'metric mass' and centre of gravity position), then all incremental measurements relative to this tare are proportional to load **on an absolute scale**. The total load, (and hence the support deflection and balance orientation), are thus available without iteration, and the component load distribution can be determined from a knowledge of the magnitude and centre of gravity location of the 'metric mass'. The concept of 'buoyant' tares thus considerably simplifies the determination of the aerodynamic loading — the objective of the test.

The component strain gauge bridge 'buoyant' offsets are hypothetical of course as the balance is never 'weightless'. However, it will be shown that they can be derived from tare data taken at different balance orientations; these data also serve to define the magnitude and centre of gravity location of the 'metric mass'. The formulation given is general and self-contained; it calculates all necessary information from the tare (wind-off) data obtained at a series of balance orientations, and hence it is unnecessary to determine the model weight and centre of gravity by separate means.

Another virtue of the 'buoyant' concept has been mentioned previously in connection with calibration of the balance; namely the avoidance of 'initial load effects' for non-linear balance systems by the use of 'buoyant' tares for both calibration and application of the balance.

4.1 Balance-Axes Load Components of the Metric Mass

In the concept of 'buoyant' tares it is necessary to consider the 'metric mass' as opposed to the weight of the model. This 'metric mass' is defined to include all parts of the model/balance system whose weight results in loads on any of the balance components, the magnitude and distribution of which depend upon the orientation of the balance relative to a gravitational system of axes. Due to the method of construction of the balance, it is possible that different components may sense different 'metric mass' contributions from parts of the balance, and therefore different total tare weights and centre of gravity positions. This will be illustrated later by the fact that, in a six-component balance for example, expressions for the weight and centre of gravity position are defined separately by the balance components measuring loads in the normal, side, and axial force planes. In what follows, 'tare weight' is used to indicate the product of the 'metric mass' and the gravitational constant.

A very common type of model support mechanism utilizes freedom in pitch (α) and roll (ϕ) to position the model at any desired orientation in the wind tunnel. The components of the tare weight, in a system of balance axes, are given in terms of these angles by:

$$W_x = W \sin\alpha$$

$$W_y = W \cos\alpha \sin\phi$$

$$W_z = W \cos\alpha \cos\phi$$

where W_x, W_y, and W_z are the tare weight components along the 'x', 'y', and 'z' axes respectively — see Figure 1.

Note that the choice of pitch and roll angles for definition of model orientation is not restrictive, as the matrix of direction cosines defining the position of the axes system can be generated by a sequence of Euler rotations. If the first rotation applied is one about the vertical axis it will have no effect on the component load distribution of the tare weight. Further rotations of pitch and roll respectively will thus provide the equivalent pitch and roll angles for the given balance orientation, regardless of the sequence of rotations used to define the orientation originally. The determination of different Euler rotation sequences, corresponding to a given matrix of direction cosines, is described in Reference 10.

The net tare loads sensed by the balance components can be represented by the vector $\underline{F}^N$ which is defined as:

$$\underline{F}^N = W\ [\underline{G1}\sin\alpha + \underline{G2}\cos\alpha\sin\phi + \underline{G3}\cos\alpha\cos\phi] \qquad (52)$$

where the elements of the vectors $\underline{G1}$, $\underline{G2}$, and $\underline{G3}$ are functions of the geometric position of the balance components and the position of the centre of gravity of the tare load W. By combining these three vectors and the weight we define a matrix [G] thus:

$$[G] = [W\,\underline{G1} \mid W\,\underline{G2} \mid W\,\underline{G3}]$$

and can now re-write Equation (52) as:

$$\underline{F}^N = [G] \begin{bmatrix} \sin\alpha \\ \cos\alpha\sin\phi \\ \cos\alpha\cos\phi \end{bmatrix} \qquad (53)$$

Extending this same notation to express the component loads for several different tare orientations we have:

$$\underset{(n,m)}{[F^N]} = \underset{(n,3)}{[G]} \underset{(3,m)}{\left[\begin{array}{c|c|c|c} \sin\alpha_1 & \sin\alpha_2 & \ldots & \sin\alpha_m \\ \cos\alpha_1 \sin\phi_1 & \cos\alpha_2 \sin\phi_2 & \ldots & \cos\alpha_m \sin\phi_m \\ \cos\alpha_1 \cos\phi_1 & \cos\alpha_2 \cos\phi_2 & \ldots & \cos\alpha_m \cos\phi_m \end{array}\right]} \tag{54}$$

where 'n' is the number of balance components, and
'm' is the number of tare attitudes.

For convenience we can denote the matrix comprising the trigonometric functions of the tare angles by [T] so that Equation (54) becomes:

$$\underset{(n,m)}{[F^N]} = \underset{(n,3)}{[G]} \underset{(3,m)}{[T]} \tag{55}$$

This equation defines the balance component loads in terms of the unknown matrix [G] and the tare attitude expressed by the pitch and roll angles. Obviously if the elements of [G] can be determined from the tare data, this relation provides the means to determine tare load effects at any attitude.

4.2 Expressions for the Elements of [G]

For a particular design of balance the elements of the matrix [G] can be defined analytically from a consideration of the static equilibrium of the system. For the case of the generally familiar type of six-component balance manufactured by the Able (formerly Task) Corporation, the component load vector is:

$$\underline{F}^N = \{N1, N2, Y1, Y2, X, R\}$$

representing the forward and aft normal and side force components, and the axial force and rolling moment respectively. The matrix [G] is given by:

$$\underset{(6,3)}{[G]} = W \begin{bmatrix} -z_o/2a & 0 & -0.5\,(1 + x_o/a) \\ z_o/2a & 0 & -0.5\,(1 - x_o/a) \\ y_o/2b & 0.5\,(1 + x_o/b) & 0 \\ -y_o/2b & 0.5\,(1 - x_o/b) & 0 \\ 1 & 0 & 0 \\ 0 & -z_o & y_o \end{bmatrix}$$

where (x_o, y_o, z_o) define the centre of gravity of the tare weight, relative to the balance reference point midway between the forward and aft components,

2a is the normal force component spacing,

2b is the side force component spacing,

and the sign convention for x_o, y_o, and z_o is positive forward, to starboard, and downward respectively — see Figure 1.

Inspection of these expressions for the elements of [G] illustrates the fact, noted earlier, that the weight and c.g. position are defined separately by different components. For example, 'x_o' is obtainable from either the normal force or the side force components, 'y_o' from either side force or rolling moment, and 'z_o' from either normal force or rolling moment. Likewise the tare weight 'W' is obtainable from either the normal force, side force, or axial force components. There is, however, no need to evaluate the weight or tne c.g. position explicitly; the objective of the tare procedure is the evaluation of the balance component load vector resulting from the tare weight, and this is given by Equation (53) in terms of the matrix [G] and the vector made up of trigonometric functions of the tare attitudes.

4.3 The 'Buoyancy' Condition

The hypothetical condition of 'buoyancy' can be defined as that condition for which the electrical outputs from the various component strain gauge bridges result only from resistive imbalances due to influences other than applied loading. Such influences would include, for example, mismatching of the individual strain gauge resistance values, strains resulting from the bonding of the strain gauges to the flexure elements, and strains arising from assembly of multi-part balances. The component outputs arising from any such 'non-load' influences will be termed the 'buoyant' component offsets, and will be, by definition, independent of any loads applied to the balance, as, for example, by variation of the balance attitude during the acquisition of tare data.

4.4 Determination of the Matrix [G] and the Buoyant Offsets

The acquisition of balance tare data consists of determining:

(a) the model/balance orientation in terms of the pitch (α) and roll (ϕ) angles common to many model support systems; alternatively equivalent pitch and roll angles can be established by computing the Euler angles in a 'yaw-pitch-roll' rotation sequence which yield the direction cosine matrix defining the orientation of the balance axes.

(b) the 'gross' outputs from each of the 'n' balance components ($\underline{m}^T$) where the measurements are made relative to the 'zero' of the measuring system.

Defining a vector ($\underline{m}^o$) to be the 'buoyant' offsets, we can express the net component outputs (which are proportional to the true tare load) by:

$$\underline{m}^N = (\underline{m}^T - \underline{m}^o) \qquad (56)$$

and this net output vector is related to the vector of true component tare loads ($\underline{F}^N$) by Equation (38). Thus:

$$\underline{m}^N = [C1]\,\underline{F}^N + [C2]\,\underline{F}^* \qquad (57)$$

assuming a non-linear balance calibration.

Pre-multiplication of both sides of this equation by the diagonal matrix composed of the reciprocals of the diagonal elements of [C1], reduces it to the form of Equation (39) in which the calibration coefficient matrices are independent of the measuring system. Thus:

$$\underline{F}^{N\sim} = [X1]\,\underline{F}^N + [X2]\,\underline{F}^* \qquad (58)$$

where

$$\underline{F}^{N\sim} = [D]\,\underline{m}^N$$

Pre-multiplication of both sides of Equation (56) by [D] gives:

$$[D]\ \underline{m}^N = [D]\ [\underline{m}^T - \underline{m}^o]$$

or

$$\underline{F}^{N\sim} = \underline{F}^{T\sim} - \underline{F}^{o\sim}$$

and thus Equation (58) becomes:

$$\underline{F}^{N\sim} = (\underline{F}^{T\sim} - \underline{F}^{o\sim}) = [X1]\ \underline{F}^N + [X2]\ \underline{F}^*$$

from which

$$\underline{F}^N = [X1]^{-1}\ \underline{F}^{T\sim} - [X1]^{-1}\ \underline{F}^{o\sim} - [X1]^{-1}\ [X2]\ \underline{F}^* \qquad (59)$$

Introducing the vector of 'buoyant' **offset loads**, i.e. the loads equivalent to the 'buoyant' electrical offsets ($\underline{m}^o$) as defined by **a particular linear calibration matrix**, we define it to be:

$$\underline{F}^o = [X1]^{-1}\ \underline{F}^{o\sim}$$

and that of the gross tare loads, which include the 'buoyant' offset loads, to be:

$$\underline{F}^T = [X1]^{-1}\ \underline{F}^{T\sim}$$

then Equation (59) becomes:

$$\underline{F}^N + \underline{F}^o = \underline{F}^T - [X1]^{-1}\ [X2]\ \underline{F}^* \qquad (60)$$

where

$$\underline{F}^* = f(\underline{F}^N)$$

Substituting for $\underline{F}^N$ from Equation (53) we have:

$$[G] \begin{bmatrix} \sin\alpha \\ \cos\alpha \sin\phi \\ \cos\alpha\cos\phi \end{bmatrix} + \underline{F}^o = \underline{F}^T - [X1]^{-1}\ [X2]\ \underline{F}^* \qquad (61)$$

The acquisition of tare data at 'm' different combinations of α and ϕ will result in 'm' equations of this form which can be expressed conveniently in matrix notation:

$$\underset{(n,3)}{[G]}\ \underset{(3,m)}{[T]} + \underset{(n,m)}{[F^o]} = \underset{(n,m)}{[F^T]} - \underset{(n,n)}{[X1]^{-1}}\ \underset{(n,p)}{[X2]}\ \underset{(p,m)}{[F^*]} \qquad (62)$$

Consider now the matrix $[F^o]$ of the 'buoyant' offset loads. Each column is composed of the vector of offset loads ($\underline{F}^o$) for a particular tare attitude which is defined as:

$$\underline{F}^o = [X1]^{-1} [D] \underline{m}^o$$

By definition the vector of the 'buoyant' electrical offsets ($\underline{m}^o$) is independent of the orientation of the balance, and thus the vector of offset loads ($\underline{F}^o$) will be independent of attitude also, provided that the linear calibration coefficients (i.e. the matrices $[X1]$ and $[D]$) are the same for all tare attitudes. However, as has been noted previously, it is quite common to calibrate a balance so as to define separate positive and negative load coefficients, and in this case there is no guarantee that the matrix product

$$[X1]^{-1} [D] = [C1]^{-1}$$

will be identical for all tare attitudes, as these may generate different distributions of the signs of the component loads. Thus the offset load vectors ($\underline{F}^o$) for each tare attitude may be different, despite the fact that the electrical offset vectors ($\underline{m}^o$) must be identical.

The formulation depends upon the various $\underline{F}^o$ being identical for all tare attitudes, a condition which clearly is satisfied in the most general case only when the 'buoyant' electrical offsets ($\underline{m}^o$) are zero. In practice this can never be assured, and in fact it is very unlikely to be the case. However, the assumption inherent in the formulation can be validated by using an iterative computation procedure in which the 'buoyant' offsets ($\underline{m}^o$) computed initially are subtracted from the tare data, which are then re-processed to provide new (smaller) offsets. This iteration can be continued until the calculated 'buoyant' offsets are zero, thus satisfying the assumption made. In this case, necessary only when separate calibration coefficients for positive and negative load are used, the actual 'buoyant' offsets would be given by the summation of all $\underline{m}^o$ determined in the iterations.

We may therefore assume that the vector $\underline{F}^o$ is identical for all tare attitudes; consequently the matrix $[F^o]$ is composed of 'm' identical columns. The addition defined on the L.H.S. of Equation (62) can thus be accomplished very simply by matrix multiplication of expanded $[G]$ and $[T]$ matrices. The $[G]$ matrix is expanded to contain four columns, the last of which is made up of the vector $\underline{F}^o$, and the $[T]$ matrix is expanded to four rows by addition of a unit row. Thus:

$$\underset{(n,3)}{[G]}\ \underset{(3,m)}{[T]} + \underset{(n,m)}{[F^o]} = \underset{(n,4)}{[G \mid F^o]} \underset{(4,m)}{\begin{bmatrix} T \\ \hline 1 \ldots 1 \end{bmatrix}} = [G1]\,[T1]$$

and Equation (62) now becomes:

$$[G1]\,[T1] = [F^T] - [X1]^{-1} [X2] [F^*] \tag{63}$$

in which $[T1]$ and $[F^T]$ are known from the tare measurements, and:

$$[F^*] = f([F^N])$$

This equation may be solved iteratively using as a starting point the solution obtained by neglecting the second order terms. Thus a first approximation of $[G1]$, or the complete solution for the case of a linear balance calibration, is given by:

$$\underset{(n,4)}{[G1]} = [\underset{(n,3)}{G} \;\vdots\; \underset{(n,1)}{F^o}] = \underset{(n,m)}{[F^T]} \; \underset{(4,m)}{[T1]^{-1}} \tag{64}$$

In order for the matrices $[F^T]$ and $[T1]$ to be conformable the number of tare attitudes 'm' must clearly be equal to 4, a condition which also satisfies the requirement that $[T1]$ be square to permit its inversion. Thus, provided that the four tare attitudes are chosen to yield a non-singular matrix $[T1]$, the required matrix $[G1]$ is readily obtainable.

We then have:

$$[G1] = [G \;\vdots\; F^o]$$

and the component tare loads at attitude (α_i, ϕ_i) are given by:

$$\underline{F}_i^N = [G] \begin{bmatrix} \sin \alpha_i \\ \cos \alpha_i \sin \phi_i \\ \cos \alpha_i \cos \phi_i \end{bmatrix}$$

The 'buoyant' electrical offsets ($\underline{m}^o$) are defined in terms of the offset loads by:

$$\underline{m}^o = [D]^{-1} [X1] \underline{F}^o$$

provided that the calibration did not involve 'plus-minus' coefficients. Otherwise it is necessary to compute the $\underline{m}^o$ vectors for each tare attitude using matrices $[D]$ and $[X1]$ whose elements correspond to the correct component load sign distribution for each tare, and subtract these 'approximate' electrical offsets from the gross output vectors $\underline{m}^T$. The resulting matrix of approximate 'net' outputs for all tares is used to compute new offset load values which are ultimately reduced to zero by iteration. At this point the necessary assumption that all offset load vectors are zero is valid. The actual 'buoyant' electrical offsets are obtained as the difference between the measured electrical outputs for one of the tare attitudes and the values of the elements in the 'net' output vector, for the same tare attitude, used in the last iteration. The solution for a linear calibration is then complete; for a non-linear calibration an iterative solution is required, using this linear approximation as a starting point.

4.4.1 Iterative Solution for Non-Linear Balances

By neglecting the non-linear terms initially, an approximation to the offset load vector is obtained — $\underline{F}^o{}_0$ say, where the subscript (0, 1, ... n) will be used to indicate successive approximations in the iteration. Thus the matrix $[F^o]_0$, having identical columns equal to $\underline{F}^o{}_0$, can be generated and:

$$[F^N]_0 = [F^T]_0 - [F^o]_0$$

where

$$[F^T]_0 = [X1]^{-1} [D] [m^T]$$

and hence a first estimate of the non-linear force matrix $[F^*]_0$ can be calculated. This gives a further approximation, $[F^T]_1$ say, to the R.H.S. of Equation (63):

$$[F^T]_1 = [F^T]_0 - [X1]^{-1}[X2][F^*]_0$$

and, from Equation (64), we then have:

$$[G1]_1 = [F^T]_1 [T1]^{-1}$$

The last column of [G1] yields a further approximation of the offset load vector $\underline{F}^o{}_1$, and hence the matrix $[F^o]_1$, and the iteration proceeds.

In general:

$$[F^T]_n = [F^T]_0 - [X1]^{-1}[X2][F^*]_{(n-1)}$$

where

$$[F^*]_{(n-1)} = f\{[F^T]_{(n-1)} - [F^o]_{(n-1)}\} = f\{[F^N]_{(n-1)}\}$$

$$[G1]_n = [F^T]_n [T1]^{-1}$$

and $[F^o]_{(n-1)}$ is composed of identical columns equal to the last column of $[G1]_{(n-1)}$.

Convergence of the solution is established when:

$$[F^T]_n - [F^T]_{(n-1)} \leqslant \underline{\delta}$$

where the vector $\underline{\delta}$ consists of specified limits for each balance component. The solution has been found to converge rapidly for typical non-linear balances, the convergence being aided considerably by the fact that the loads involved in tare measurements are normally small relative to the balance rated loads. Thus non-linear effects will be quite small even when the non-linear calibration coefficients are relatively large. As a precaution against divergence of the solution, the values of $[F^T]_n$ may be compared with test values related to the component load capacities.

As in the case of the linear solution, when 'plus-minus' balance calibration coefficients are employed it is necessary, upon convergence of the non-linear solution, to iterate the entire procedure in order to reduce the offset load vector ($\underline{F}^o$) to zero.

4.5 Application of the Tare Procedure

The objective of the tare computation is the evaluation of the matrix [G] in Equation (53) and the vector of 'buoyant' electrical offsets $\underline{m}^o$, both of which are defined by the matrix [G1] determined by the procedure given. By using the 'buoyant' offsets ($\underline{m}^o$), which are defined in the measurement units of the data acquisition system, all loads are automatically computed relative to absolute zero load, and thus support rotations and the deflected orientation of the balance axes system are readily obtainable. Determination of the equivalent pitch and roll angles, (as the second and third Euler rotations in a 'yaw-pitch-roll' sequence of rotations which yields the direction cosine matrix of the required axes orientation), then enables direct use of Equation (53) to establish the balance component loads of the tare weight. These are subtracted from the total component loads to arrive at the required aerodynamic loading.

It is clearly unnecessary to evaluate the tare weight and centre of gravity position explicitly. However, as indicated in Section 4.2, the elements of the matrix [G] can be defined analytically in terms of W, x_o, y_o, and z_o for any particular balance, from which it is possible to evaluate these quantities by substitution of the numeric values of the matrix elements. To do so often provides a useful 'diagnostic' indication of the validity of the data; obviously incorrect results usually indicate a violation of the assumption that only gravitational forces are acting on the balance, perhaps as a result of 'fouling' between the 'live' and 'ground' sides of the model/balance system.

In situations combining a relatively heavy model and a relatively flexible support, it may be desirable to correct the tare attitudes for rotations of the support under load. Usually the support stiffness constants will be defined in terms of the balance component loads, and thus the rotations are easily computed from the balance components of the tare load. Following application of the tare procedure to evaluate the matrix [G], the balance tare components can be determined from Equation (53), the support rotations calculated, and the tare attitudes corrected accordingly. The tare procedure can then be applied again, using the corrected attitudes, and [G] and $\underline{m}^{o}$ obtained. Such a correction for support rotations is essentially exact, as the change in the computed component tare loads as a result of the deflection will be very small compared to the magnitude of the component loads.

4.6 Optimization of the Tare Attitudes

As described, the tare procedure requires data to be recorded at four unique model/balance attitudes, though note here that it is the orientation of the balance axes system rather than the model which is important. This represents the most general case where the balance system is responsive to all three components of the tare weight, (W_x, W_y, and W_z); abbreviations in which one or more of these components do not influence the balance readings will be discussed later. The question which now arises is, what combination of the four tare attitudes will provide the most accurate result?

A possible answer is suggested by observation of Equation (64) which states:

$$[G1] = [F^T]\,[T1]^{-1}$$

It is clear that the combination of attitudes must result in a non-singular matrix [T1], which consists of trigonometric functions of the pitch and roll angles defining the balance orientation. Further, as the matrix $[F^T]$ is composed of measured balance data which are subject to experimental error, minimizing the absolute values of the elements of the inverse of [T1] might logically be expected to minimize the effect of these experimental errors following the multiplication indicated. This condition would be equivalent to maximizing the absolute value of the determinant of the matrix [T1].

Using this criterion, the optimum combination of tare attitudes,

$$[(\alpha_i, \phi_i), i = 1,4]$$

were determined using an existing computer program based on the Simplex method[11]. The result obtained is summarized below:

Tare # (i)	α_i	ϕ_i
1	α_1	ϕ_1
2	α_1	$(\phi_1 + 180°)$
3	α_2	$(\phi_1 + 90°)$
4	α_2	$(\phi_1 + 270°)$

The value of ϕ_1 is arbitrary, the maximum absolute value for the determinant occurring when:

$$\alpha_1 = -\alpha_2 = \sin^{-1}(1/\sqrt{3}) \simeq 35.3°$$

and

$$|\Delta| = 16/(3\sqrt{3}) \simeq 3.08$$

Using this combination of attitudes, the analytic expression for the determinant simplifies to:

$$\Delta = 4 \cos\alpha_1 \cos\alpha_2 (\sin\alpha_1 - \sin\alpha_2)$$

Figure 2 illustrates the variation of $|\Delta|$ with the total range of the pitch angle (α), and the mid point of this range.

It is suggested that the arbitrary datum roll angle (ϕ_1) should be chosen such that all balance components influenced by changes of roll angle should experience finite loads at all tare attitudes. This will ensure a unique choice of 'plus-minus' calibration coefficients for the component load sign distribution of each tare attitude. A datum roll angle of 45 degrees, combined with as large a range of the pitch angle as practicable (up to the optimum value indicated above), would therefore be appropriate.

4.7 Abbreviations of the General Case

Situations can arise in which one or more of the balance-axes tare weight components, (W_x, W_y, and W_z), may produce no variation of any balance component load as the model/balance attitude is changed. Such would be the case, for example, with a balance designed to measure axial force only, as here the components W_y and W_z have no influence on the balance output. In such cases Equation (53) can be abbreviated by elimination of the appropriate balance-axes tare load component, or components.

Another situation requiring abbreviation of the general formulation arises when the model attitude variation is restricted to either pitch or roll for some practical consideration. For example, should there be no capability to alter the roll angle, it would not be possible to acquire four independent tares and the result would be a singular matrix [T1]; here the elements of the second and third rows, (i.e. for W_y and W_z), would not be independent. To obtain a non-singular matrix [T1] in this case, it is necessary to eliminate either W_y or W_z from the formulation; each balance-axes tare component eliminated reduces the required number of tares by one.

There are consequently two situations requiring the use of an abbreviated form of the formulation. In the first case described, there is no loss of generality as the balance-axes components are eliminated only because they do not affect the balance measurements. However, in the second case the component must be eliminated as it is not independent. For whichever reason, elimination of a component brings about a contraction of the [G] matrix, the contracted form correctly relating the balance tare components and the balance-axes (gravity) components of the tare load only when the attitude is compatible with the tare attitude. Thus, if the tare data are recorded at constant roll angle, the contracted [G] matrix calculated by the tare procedure will give correct balance tare components only at that constant roll attitude.

The most general tare condition is that given in Equation (52); the six possible abbreviated forms are noted below:

(a) W_x eliminated: $\quad \underline{F}^N = W[\underline{G2} \cos\alpha \sin\phi + \underline{G3} \cos\alpha \cos\phi]$

(b) W_y eliminated: $\quad \underline{F}^N = W[\underline{G1} \sin\alpha + \underline{G3} \cos\alpha \cos\phi]$

(c) W_z eliminated: $\underline{F}^N = W[\underline{G1} \sin\alpha + \underline{G2} \cos\alpha \sin\phi]$

(d) W_y and W_z eliminated: $\underline{F}^N = W[\underline{G1} \sin\alpha]$

(e) W_x and W_z eliminated: $\underline{F}^N = W[\underline{G2} \cos\alpha \sin\phi]$

(f) W_x and W_y eliminated: $\underline{F}^N = W[\underline{G3} \cos\alpha \cos\phi]$

Forms (a), (b), and (c) require three tares while (d), (e), and (f) require two tares only.

Generally the latter three abbreviations would be used when the balance was influenced by one gravity component only, and thus are particular to balances designed to measure loads in just one plane. Cases (a), (b), and (c) represent the more interesting situation brought about by physical restrictions in the model attitude control.

Case (a) corresponds to all tares being acquired at constant pitch angle. The contracted [G] matrix contains two columns, appropriate to the gravity components W_y and W_z, which correctly define the balance tare components **at the given constant pitch angle only**. However, the computed electrical offsets ($\underline{m}^{\circ}$) will be 'buoyant' only if the constant pitch angle is 0°. At any other constant value the **offset loads** calculated for each balance component will be the sum of the true 'buoyant' offset load and the component load generated by the 'W_x' component of the tare load. The offset load for rolling moment will consequently be 'buoyant' regardless of the value of the pitch angle, and, knowing the tare weight and centre of gravity location, the true 'buoyant' offsets for the other components could be determined if desired. A similar situation exists for the abbreviated cases (e) and (f), while for case (d) the calculated offset for the 'X' balance component is 'buoyant'.

Cases (b) and (c) both correspond to the acquisition of all tares at constant roll angle. A roll angle of 0 or 180 degrees requires the use of case (b), while case (c) is appropriate if the angle is 90 or 270 degrees. For any angle other than those noted either abbreviation may be used, although the logical choice would be to eliminate the gravity component exhibiting the smaller variation with pitch angle. For these cases the offsets computed are the true 'buoyant' offsets, regardless of the value of the constant roll angle.

The optimum tare attitude combinations for these abbreviated cases, again based on maximizing the absolute value of the determinant of [T1], are given below:

Case	α_1	α_2	α_3	ϕ_1	ϕ_2	ϕ_3	$\lvert\Delta\rvert$ max at
(a)	0°	0°	0°	ϕ_1	0°	$-\phi_1$	ϕ_1 = 120°
(b)	α_1	0°	$-\alpha_1$	0°/180°	0°/180°	0°/180°	α_1 = 120°
(c)	α_1	0°	$-\alpha_1$	90°/270°	90°/270°	90°/270°	α_1 = 120°
(d)	α_1	$-\alpha_1$	...	ϕ_1	ϕ_1	...	α_1 = 90°
(e)	0°	0°	...	ϕ_1	$-\phi_1$	...	ϕ_1 = 90°
(f)	0°	0°	...	ϕ_1	ϕ_1 + 180°	...	ϕ_1 = 0°

5.0 CONCLUDING REMARKS

This report has discussed the calibration and application of strain gauge balances with particular reference to their use in wind tunnels. The procedures for multi-component loading calibrations and the application of force balance tares utilizing the concept of 'buoyancy', were developed at NAE as part of a program aimed at rationalizing the balance calibration and data reduction procedures used in connection with tests performed in the 5 ft. × 5 ft. Wind Tunnel and the 15 in. × 60 in. Two-Dimensional Test Section. The procedures described have been in use at NAE for a considerable period and have proven to be very satisfactory. Procedures for solution of the second order balance equations have been included to make this report a fairly complete description of methods for calibration and use of strain gauge balances. However, the non-iterative method outlined is not employed at NAE for reduction of balance data from the 5 ft. × 5 ft. Wind Tunnel.

A general method for the calibration of multi-component strain gauge balances has been presented, and compared with the common procedure in which the components are loaded independently and constant secondary loads are used to determine the non-linear interactions. In presenting both methods the equations defining the balance behaviour have been limited to second order terms as, from the experience of many users, higher order effects have been shown to be negligible for typical balances. However, both methods are easily extendable to include higher order effects if desired.

A major difference between the two methods is that the latter specifically limits the order of the interaction terms, usually to second order, by restricting the number of component loads acting simultaneously, whilst the former allows the existence of higher order effects but limits the terms in the balance equations by the choice of the order of the polynomial to which the data are fitted. Any terms in the general balance equation which are deemed to be negligible in a particular situation may be eliminated, (the interaction of a small axial force on a large normal force for example), thus abbreviating the equations and hence the number of loading cases necessary to obtain a solution. Note however that, if the component loads involved remain non-zero during the calibration process, (normal and axial forces in the example), elimination of the term from the equation does not remove the actual interaction, and the abbreviated formulation still represents a best solution of the equations when using the actual data.

The proposed method allows standardization of calibration procedures for both 'internal' and 'external' types of balance. The use of a single varying calibration load permits the use of loadings which are representative of those to be expected in a test environment, thus ensuring reasonable similarity of the deflections occurring in both calibration and use of the balance. All calibration coefficients can be obtained without the need of 'secondary' loads, thus simplifying the calibration set-up. The formulation does, however, permit the inclusion of constant 'secondary' loads on one or more components, thus providing a means of accounting for the weight of the calibration body so that the load on each component may be specified relative to absolute zero load — i.e. a 'buoyant' zero. In combination with the method for defining 'buoyant' tares, this avoids 'initial load effects' in the use of non-linear balances.

The choice of a 'linear' versus a 'non-linear' calibration should be based upon whether or not the simpler linear matrix can provide adequate accuracy for the particular test requirements. With the multi-component loading technique a linear solution is really a linearization of the complete balance behaviour. This may provide a better accuracy than the linear solution obtained using the technique of loading the components independently, as in this case no non-linear effects are present during the calibration. When 'linearizing' a non-linear system it should be noted that the linear curve fit of the data should be forced to pass through the actual data value at zero load; performing the curve fit with a 'free' intercept will result in a uni-directional bias upon subtraction of 'tares'.

The ability of the derived matrix of sensitivity and interaction coefficients to adequately represent the actual balance behaviour should be verified, by using it to re-calculate the applied load for some check cases which closely simulate the expected test loadings. A convenient means of expressing the result is to compute the percentage error in the magnitude of the load vector, and the angular error in its direction.

The most satisfactory representation of the balance behaviour will probably be obtained by treating the response to positive and negative loads separately. It is, however, most important to recognize that it is the sign of the strain in the flexure element which matters, not the sign of the load for some 'derived' component which is a function of two or more flexure loads. It seems superfluous to state this, but the author is aware of a case in which the behaviour of a 6-component balance was specified by all linear, load squared, second order load cross-product, load cubed, and load 'absolute value' terms; however the absolute value terms were based on the signs of derived component loads, a fact which considerably undermined any confidence which might have been placed in the inclusion of the load cubed terms!

A non-iterative method for solution of the second order balance equations has been presented. However, it is felt that the approximation involved cannot be justified, as the iterative solution converges rapidly for typical balances. Moreover, the non-iterative method cannot accommodate the use of separate calibration coefficients appropriate to positive and negative component loads. Two methods of accounting for such 'plus/minus' calibration matrices have been described. One method involves the synthesis of a specific matrix, appropriate to a given vector of component loads, from the general 'plus/minus' matrix. This specific matrix must be inverted as part of the solution for the component loads. The second method uses pre-defined coefficient matrices relating to the 'signed' and 'absolute' values of the component loads, and in this case no matrix inversion is required although additional matrix multiplications are necessary. From a computational efficiency standpoint, the latter method is to be preferred.

The procedure described for the application of balance tares uses a concept of 'buoyancy' to overcome the iterative nature of the problem; this arises if the attitude is not sensed within the model and must therefore be corrected for support deflections under load. Additional advantages of the method are the determination of the 'buoyant' electrical offsets for each component, which provides a convenient means of monitoring the long-term stability of the balance, and the fact that the derived weight and centre of gravity information can provide a useful 'diagnostic' of the operation of the balance — e.g. freedom from 'fouling' between the 'live' and 'ground' members. The method requires measurement of the component outputs and the balance orientation at a maximum of four independent positions. The orientation is defined in terms of the pitch and roll angles available with a common type of model support system, and optimum combinations of these angles are suggested. For support systems which do not utilize the specified pitch and roll angles for model positioning, the necessary angles can be determined from the matrix of direction cosines which defines the orientation of the balance axes system.

The use of the 'buoyant' tare approach, in combination with a definition of loads on an absolute scale during balance calibration, avoids the problem of initial load effects in the use of non-linear balances.

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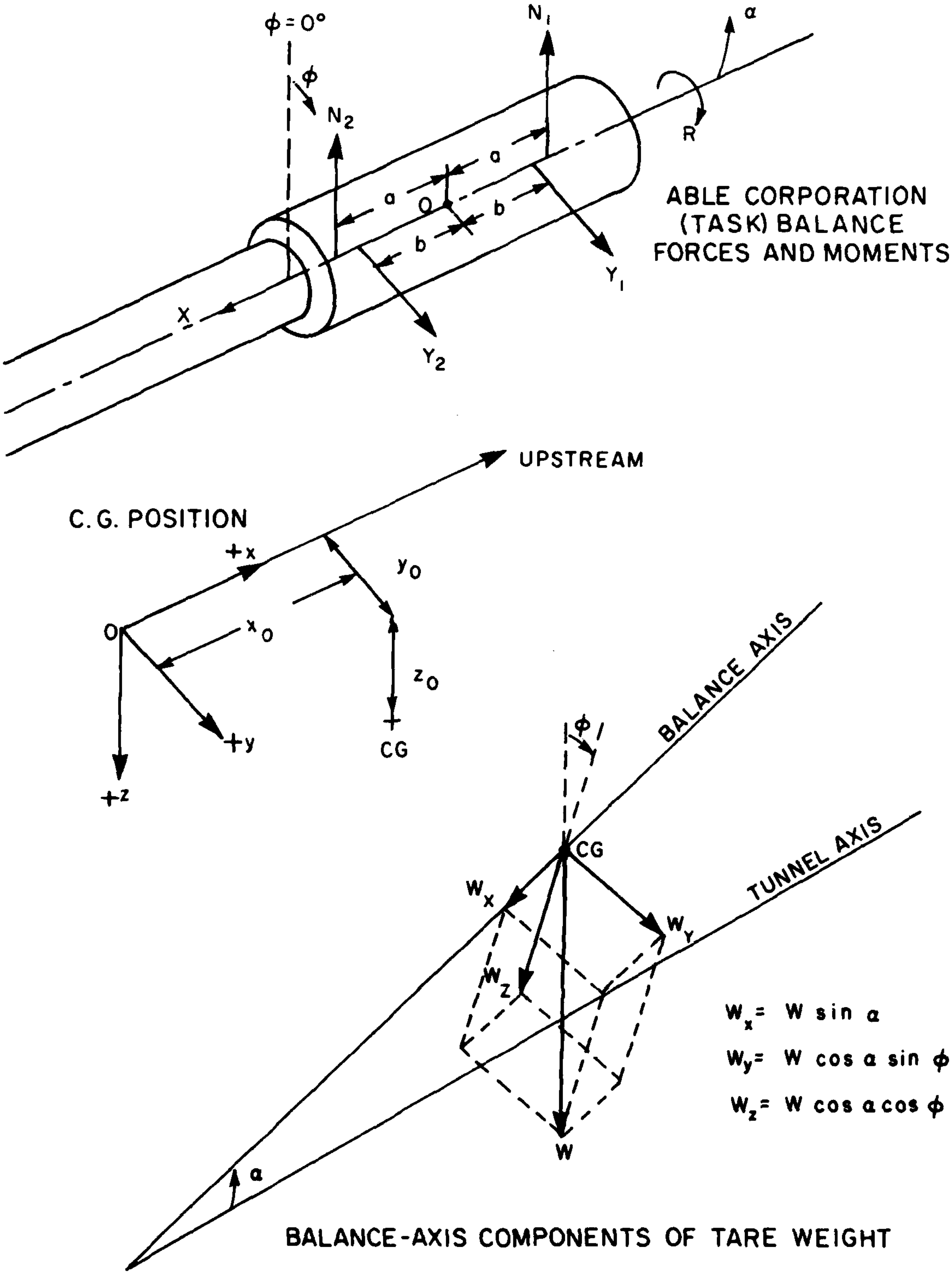

(POSITIVE DIRECTIONS ARE INDICATED BY ARROWS)

FIG. 1: SIGN CONVENTIONS FOR FORCES AND DISTANCES

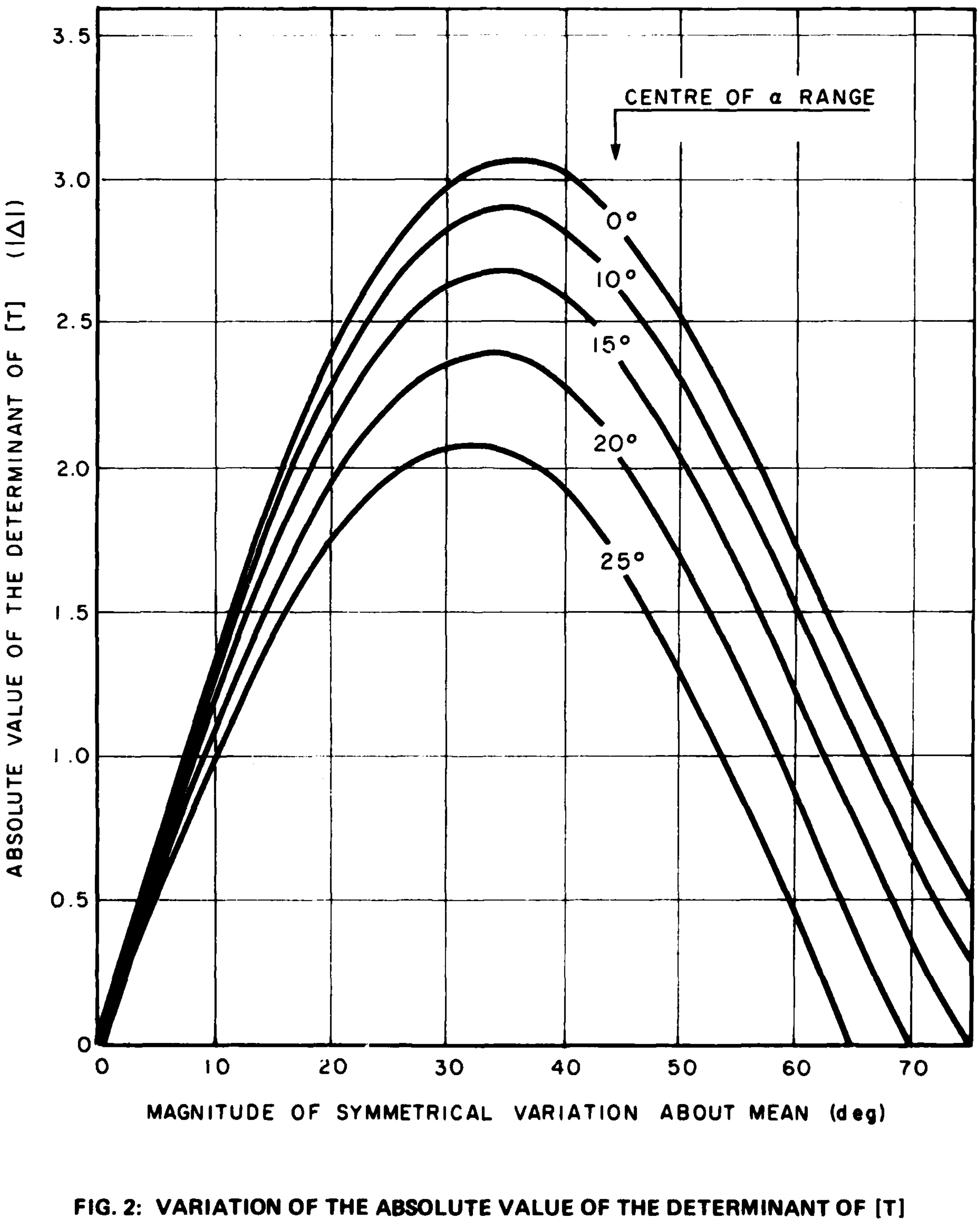

FIG. 2: VARIATION OF THE ABSOLUTE VALUE OF THE DETERMINANT OF [T] WITH PITCH ANGLE (α) FOR THE OPTIMIZED ATTITUDES

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The usual iterative solution of the second order balance equations is outlined, and an approximate non-iterative scheme is included for completeness, though not recommended. Two methods of accounting for dependency of the calibration coefficients on the signs of the component loads are presented.

A concept of 'buoyancy' is introduced to simplify the application of force balance tares, and a procedure for determining the component outputs for absolute zero load (the 'buoyant' offsets) is given. Balance data at a series of model attitudes are used to define these offsets, and also the coefficients in the equations defining the component load distribution of the tare weight at any attitude.

The topics covered are ideally suited to formulation and solution by matrix methods, which have been used throughout.

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1. Strain gauge balances.

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